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# ORDERLY MARKETING <br> FOR <br> CALIFORNIA AVOCADOS 

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This study describes the salient features of avocado marketing - sources and trends of supply, concentration and collaboration among handlers, heterogeneity of the commodity, seasonality and storeability, handling costs, and selling policies. In addition, quantitative characteristics of the wholesale demand for Calavo avocados are estimated, both by years and by weeks. Finally, the study develops a procedure for planning sales within a season to the best advantage of California growers and handlers.
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## ORDERLY MARKETING FOR CALIFORNIA AVOCADOS ${ }^{1}$ STEPHEN H. SOSNICK ${ }^{2}$

## INTRODUCTION

Over 100 varieties of avocado are produced in California. The Fuerte accounts for about 60 per cent of the tonnage, the Hass and MacArthur combined, for about 25 per cent. While avocados mature throughout the year in California, maturities are uneven, and gluts occur each spring.
The present study attempts to develop a procedure for allocating a season's sales of California avocados over the season at optimum advantage to the growers and handlers concerned. Under a marketing order, the procedure could be applied to total California output. As presented here, however, the recommendations apply specifically to the Calavo Growers of California, a marketing coöperative that handles about half of the state's crop.
Adjustments in allocation of the season's output are possible. Calavo has substantial control over harvesting. Furthermore, avocados may be stored on the trees and in the warehouse for up to five months beyond maturity. However, intraseasonal adjustment must be attempted in the face of considerable uncertainty about maturities, yields of specific picks, total production, future supply response, the relation of handling costs to quantity sold, and the prices at which different quantities would move.
California production in the 1950's had grown to about 70 per cent of United States supply; Florida production had grown to about 20 per cent; and Cuban shipments had declined to about 10 per cent. From each of these sources, avocados were shipped to all parts of the country. Calavo's share of national tonnage, including consignments of Cuban avocados in summer and of Florida avocados in autumn, was about 50 per cent. None of the remaining 60 or so handlers accounted for as much as 10 per cent.
Virtually all avocados are used fresh, primarily as a salad fruit. Because of varietal differences, California avocados have sold at higher prices per pound than Florida avocados, and the latter for more than Cuban avocados. Calavo's grading, packing, sales promotion, and dealer relations have helped Calavo avocados to command a premium over other California avocados

[^0](although it is uncertain whether the premium is worth the costs associated with obtaining it).
Calavo sells exclusively by private negotiations that are related to Calavo's list prices, f.o.b. Los Angeles. Asking prices are adjusted frequently, and can be regarded as dependent on the quantities to be sold. A satisfactory expression was obtained for the relation of actual annual average Calavo price to total California volume and nonagricultural personal income. (NonCalifornia volume and other variables were deleted after failing tests of significance.) Demand at average levels appears elastic with respect to both volume and income. For each additional million California pounds sold per year, Calavo's selling prices decreased an average of 0.3 cent per pound.
Two approaches to the measurement of weekly demand were used. They produced a satisfactory expression for the relation of weekly average Calavo price to total California quantity, total non-California quantity, nonagricultural personal income, and week of the season. There is strong evidence that demand systematically fluctuates during the season, presumably because of changing availability of other fruits and vegetables. In September, Calavo's selling prices decreased an average of 0.3 cent per pound for each additional 10,000 California pounds sold per week. By March, the average decrease was only 0.1 cent per pound.

On the basis of information about weekly demand and handling costs, the optimal intraseasonal allocation of any given total volume can be estimated. Because demand and time preference factors vary over the season, optimal allocation entails price fluctuation. The solution involves selling a quantity each week that will cause marginal net income in all weeks to be equal. If Calavo had accomplished this equality while selling its 1958-1959 volume, its seasonal income would have been about $\$ 300,000$, or 8 per cent, or 0.7 cent per pound, greater. The solution, furthermore, appears to have been consistent with maturity and storage limitations.

The 1958-1959 volume, however, was slightly larger than the 45 million pounds that could profitably have been sold fresh. Income could have been increased still more by processing, carry-over, or abandonment of about one million pounds. This diversion, however, would have increased members' returns only about 0.007 cent per pound, while increasing independents' returns about 0.3 cent per pound.

Optimal allocation of even the actual volume, however, could have been accomplished only if that volume had been planned from the start. In fact, only an uncertain prediction of total fresh volume was available at the start of the season. A tentative conclusion is that weekly quantities should have been planned as if no uncertainty attached to the prediction. Planning for a volume somewhat different from that predicted apparently would not substantially have increased expected income or reduced risk, and improvement in either one could have been obtained only at the expense of the other.

Calavo's initial prediction of 1958-1959 volume proved substantially low. Even with that prediction, however, and even with maturity and storage limitations, improved allocation was possible. Seasonal income apparently
could have been increased by about $\$ 100,000$, or 3 per cent, or 0.2 cent per pound. These figures may be taken as an estimate of the increase in 19581959 revenues available from more refined planning techniques. The implication is that Calavo did remarkably well, but might be able to do still better.

For the future, there is little doubt that it would pay Calavo to arrange for the precise programming of intraseasonal allocation. Procedures to accomplish that programming are outlined in the final section.

## MARKET STRUCTURE AND CONDUCT

## Sources of Supply

The avocado (Persea americana) is a tender, subtropical fruit. Commercial production is limited to regions of favorable soil and water conditions, mild winters (above $25^{\circ} \mathrm{F}$ ), sufficient mean temperatures (above $55^{\circ} \mathrm{F}$ ) during blossoming and fruit setting for a given variety, no sudden heat waves, appropriate total heat and humidity, and adequate protection from wind (Hodgson, 1947, p.11). ${ }^{3}$ These requirements have confined commercial production in the United States to belts centering on Los Angeles, California, and Miami, Florida, plus small amounts in Hawaii and in the Rio Grande Valley of Texas. Hawaiian production has been approximately one million pounds per year since 1947 (Hawaii Agr. Ext. Service, 1959, p. 45). After they are packed, California and Florida avocados are shipped by rail and, increasingly, by truck to all parts of the country-including each other's home territory. A small amount goes to Canada, and occasional shipments go overseas. The geographic destinations of Calavo's sales have varied considerably from season to season and from month to month, but representative figures for recent years would be 25 per cent for Los Angeles, 20 per cent for other California areas, 30 per cent for other western areas, and 25 per cent for east of the Mississippi.
In addition to those produced domestically, avocados have also been imported. Imports came predominantly from Cuba. Small-indeed, negligible -amounts also arrived sporadically from Bermuda, Chile, Dominican Republic, Haiti, Mexico, Nicaragua, Panama, and the West Indies. The quantities were small primarily because of an import duty set at 15 cents per pound in 1930 and at 7.5 cents in 1947 (U. S. Dept. Agr., 1951)-amounts that usually exceeded the potential value of the fruit. Cuban shipments were larger primarily because exempt from this duty, under the Reciprocity Treaty of 1902. Most Cuban shipments arrived in southern Florida (U. S. Dept. Agr., Foreign Agricultural Trade), usually after a 30 -hour trip by rail car ferry. They were often reloaded into trucks. Like Florida shipments, they might move anywhere in the country, but most were consumed in southern communities and in New York City. ${ }^{4}$

[^1]Table 1
ANNUAL FRESH VOLUME BY SOURCE AND CALAVO PRICE, 1925-1959


## Trends in Supply

Fresh sales, source of fruit, and Calavo's price and percentage of volume are shown in table 1. Information before 1924-1925 was not available. Data are presented by seasons that correspond to what is regarded as the California crop year-October through September. Figures for California sales were obtained from the U. S. Department of Agriculture (1948, 1952, 1956, 1957, 1958a, 1960). Figures for Florida represent data from the same publications, adjusted to the California crop year according to the author's estimates of the seasonal distribution of Florida sales (see below). Figures for imports are also derived from the U. S. Department of Agriculture (1928, 1932, 1934, 1936; and Foreign Agricultural Trade). The U. S. Department of Commerce has also published data on imports (in recent years through the Bureau of the Census), but the USDA series is apparently not only the longest but also the most reliable. However, it, too, had to be adjusted to the common crop year, and for this purpose a seasonal distribution based on data of the Department of Commerce (1937, 1938, 1940, 1941, 1944-1960) was used.

Annual supplies vary considerably, principally because of weather conditions (including hurricanes) and a tendency of avocado trees to alternate heavy and light crops. (This tendency is related to exhaustion of organic materials, and although never reliable, it has been less pronounced in recent years because of careful selection of varieties and parent trees.)

Behind the fluctuations is a roughly level over-all trend for imports and, in contrast, sustained growth in domestic production. These trends brought California and Florida sales up from negligible proportions in the early years of the century to about 70 and 20 per cent, respectively, of the total in the 1950's. Judging by information on nonbearing acreage and yields, the growth in domestic production will continue in the next several years. However, grower returns in 1957-1958 and 1958-1959 fell to levels reminiscent of the 1930's (about 9 cents per pound for Calavo members) and a readjustment may lie ahead unless the disappearance of Cuban supplies reverses the trend.

Hodgson (1947, p. 89) estimated that the area of potential avocado culture is 40 thousand acres in California and 20 thousand acres in Florida. California acreage as of 1959 was 26,055 (California Crop and Livestock Reporting Service, 1960, p. 21). Florida plantings as of 1958 were estimated at 10,500 acres, of which perhaps 300 acres were nonbearing (Ruehle, 1958, p. 6). Of the total California acreage, almost a fifth was nonbearing (planted less than five years previously). On the other hand, only 187 of those 4,750 acres had been planted in 1959, presumably a reflection of the low prices that had prevailed during 1957-1958 and 1958-1959.


## Concentration and Collaboration Among Handlers

When Calavo was organized in 1924, its 100 members accounted for 80 per cent of reported California production. During most of the 1950's, Calavo handled about 60 per cent of the California crop (table 1). Calavo's sales represented the production of 3,000 to 5,000 growers out of a total, in 1959, of 9,000 (or 6,000 , if we neglect "backyard producers" of less than 1,000 pounds per year).

Calavo's share of California production fell to 45.1 per cent in 1958-1959. This unprecedented trough reflected the withdrawal in October, 1958, of 10.5 per cent of Calavo's membership, including 23.7 per cent of members who had averaged more than 5,000 pounds each during 1956-1957 and 1957-1958. Many of those who resigned indicated that they felt Calavo's returns were not competitive. Withdrawals in October, 1959, fell to 2.2 per cent of the membership. This was the lowest rate in many years, and was outweighed by new memberships. Calavo's share of California production during 1959-1960, consequently, rose to about 49 per cent.

Calavo's share of national sales has been larger than its proportion of California tonnage suggests. Its eastern offices have handled Florida avocados since 1932, and Cuban avocados since 1946, on a consignment basis. Both arrangements were discontinued in 1958, but Florida arrangements were reactivated during 1960 at the request of Florida's major avocado and lime shipper. Sales data for recent years indicate that Calavo sold about 40 per cent of Florida production and 20 per cent of imports, for a total share of about 50 per cent of the market during the 1950's (table 1).

In addition to Calavo, there were, in 1959, 45 other California handlers (packers and fruit buyers), one of them also a coöperative. Their fruit, like Calavo's, is shipped all over the country, but a much larger proportion remains on the Pacific Coast and especially in California. Compared with Calavo, these other handlers are small. As the concentration curve in figure 1 shows, in 1958-1959, a year of minimum share for Calavo, the largest handled less than one-sixth as much tonnage.

Beside coöperatives, several other forms of joint action have been undertaken in California. In 1915 the California Avocado Society was organized. Its focus, however, is on research into production problems. Before Calavo was organized, however, the Society actively sought solutions to marketing problems. In 1921, it tried to arrange for selling avocados through the marketing coöperative now called Sunkist Growers. Failing this, it arranged a contract with a commission house, and several years later created Calavo (see yearbooks of the Society, and Stokdyk, 1932). In mid-1959 a number of packers formed the California Avocado Development Organization, a trade association with the primary function of joint sales promotion. CADO became inactive when a signup target of 90 per cent of volume was not quite reached, but was reactivated in late 1960 with revised objectives-pertaining to labor supplies, maturity control, and market research. Earlier in 1960 a
state marketing order was issued, containing authorization for sales promotion and market research (California Dept. of Agriculture, 1960). As of 1961, the order had not gone into effect because of litigation pertaining to the eligibility of "backyard producers" to participate in the referendum.

Since the numerous "independent" California handlers are not large individually, and apparently are making decisions independently, they probably will not retaliate collectively if Calavo revises its intraseasonal allocation of sales.

The Florida crop "is marketed by some 10 or 12 shippers, for the most part, ranging from growers who ship only what they produce to shippers who grow nothing. [None] handles more than 35 per cent of the crop" (Ruehle, 1958, p. 72). A multicommodity trade association exists for growers and shippers of tropical fruits and vegetables. It does not, however, undertake marketing functions. Also, since 1954, a federal marketing order has been in effect, pertaining to maturities, grades, sizes, containers, packs, movement reports, and research, but not to quantities (except for culling) or to sales promotion (Avocado Administrative Committee). (Maturity and grade regulations under this order have also been applied to imported fruit.) For present purposes, consequently, Florida shippers, too, may be regarded as a group of individual sellers who will not collectively retaliate if Calavo revises its intraseasonal allocation. As stated, none of these shippers accounts for more than 35 per cent of Florida's 25 per cent contribution to the national supply, and Calavo sold about 40 per cent of total Florida production.

## Usage

The data on total availability-less than 1 pound per capita per year-indicate that consumption of avocados cannot be both widespread and frequent. In fact, it is neither. Although domestic commercial production dates from the turn of the century, one American homemaker in eight apparently has never heard of avocados, and only one in four serves them, usually infrequently. Usage is average in the South, low in the North ( 15 per cent), and high in the West ( 60 per cent) (U. S. Dept. of Agriculture, 1958, p. 100). The (long-run) explanation may lie partly in the distinctive flavor of avocados, partly in the difficulties of selecting and using them, and principally in their retail price. At, for example, 40 cents per pound, they are often regarded as exotic luxuries. Consumption is substantially confined to the homes and restaurants of upper-income city dwellers, where avocados are used principally sliced as a salad fruit. In addition, they are often eaten from the shell, alone or with poultry or shellfish, and often made into a dip or spread.

Avocados have also been processed into oil and meal. While the oil lends itself well to various commercial uses, competition from vegetable oils has made this unprofitable except for small amounts of low-grade avocados. Dehydration, canning, and processing for use in ice cream, bread, etc., have also appeared unpromising. For present purposes, then, fresh sales are the relevant disposition.

Since there are many other salad ingredients and since avocados are used in several kinds of salads, it is unlikely (although not, of course, impossible) that the demand for avocados is strongly influenced by the availability of other commodities individually. It is even unclear whether other salad ingredients are competitive or complementary (in use, not in store space)-if indeed given commodities remain one or the other over different months, years, and areas (Talbot, 1954, p. 2). Since these ingredients are numerous, individually unimportant, divergent in movement, difficult to aggregate, and not susceptible to a priori constraints, their inclusion in a demand function may be impracticable.

With respect to the income variable in such a function, apparently it may best refer to incomes throughout the country, but particularly to city dwellers. And with respect to elasticities of demand, the continuing novelty of the commodity, its historic price level, and its use in high-income households and restaurants should prepare us for the conclusions that demand (even f.o.b. the packing house) is elastic at average values with respect to both price and income.

## Heterogeneity

The principal causes of product heterogeneity are trade mark, variety, quality, and size.

Sales Promotion. Various handlers affix names to their containers and to the individual avocados. Only Calavo, however, undertakes substantial advertising and trade promotion on the selling side, a policy which originated when Calavo's task was viewed as the creation of a western market for avocados. Calavo's promotion costs averaged about $\$ 200,000$ per year over 1957-1958 and 1958-1959, or about 2.5 per cent of its sales of California avocados (or 1.5 per cent of its sales including companion tropical fruits).

This promotional effort, plus Calavo's grading, packing, and dealer relations, has helped Calavo fruit to command a premium over other California avocados. This is true even though, according to trade information, independent fruit is not significantly different physically from that received by Calavo, and even though confidential surveys indicate that only one western housewife in 10 recognizes that "Calavo" is a brand name, and only one in 50 realizes that it is reserved for high-quality avocados.

It would be very difficult to judge how much of the price premium that accrues to Calavo is a result of its being a market leader whose prices are shaded by smaller competitors, how much is a result of special marketing expenses that Calavo incurs, and how much is the result of other factors. It would also be difficult to separate the costs to Calavo that are specifically associated with obtaining the premium. Consequently, no attempt will be made to judge whether the premium is worth the cost of obtaining it, although the question would be an important one in any over-all appraisal of Calavo's selling policies.

Existence of the price differential also suggests that it would be appropriate to include in the demand function a variable to represent sales promo-
tion. This, also, will not be done, because of the usual difficulties: dollar outlays lump together varying forms of promotion, along with home-office expenditures; outlays at different price levels are hard to deflate; advertising has been limited to selected areas; and effects presumably extended into later periods. We can, however, at least attempt to distinguish Calavo and "independent" supplies despite their physical similarity.

Varieties. Over 200 named varieties of avocado are being marketed. They have been grouped into three races-the Mexican, the Guatemalan, and the West Indian, each of which apparently originated in Central America-plus hybrids. Climatic conditions and cultural selection have determined their distribution. Cuban avocados are almost all West Indian and often wild. In Florida, West Indian varieties, formerly predominant, have yielded to Guatemalan-West Indian hybrids. California production consists mostly of Guatemalan varieties and Mexican-Guatemalan hybrids.

Each race has a different frequency distribution for fruit characteristics, such as color, texture, and thickness of skin; size and shape of fruit; size of seed; time of maturity; and percentage oil content (a principal determinant of palatability). Also, within each race, there are important differences in the characteristics of individual varieties, as well as characteristic differences in the varieties' importance. General racial differences help to explain why California avocados historically have wholesaled in comparable circumstances at a higher price per pound (although not necessarily per unit of fruit) than Florida avocados, and the latter have, in turn, brought more than Cuban avocados. Relative prices have varied considerably, but representative indexes, based on Calavo Growers' records, might be 100, 75, and 50.

The varietal composition of California production is of particular interest. The Fuerte, a Mexican-Guatemalan hybrid introduced into California in 1911, has been the leading variety since 1927. At the peaks (1936-1937, 1945-1946, 1951-1952), Fuerte constituted 85 per cent of Calavo members' tonnage, and presumably about the same proportion of California production as a whole. In the last several years, the figure has been approximately 60 per cent, as production of certain other varieties has gained momentum because of yield and hardiness. Most notable of the latter are the Hass and MacArthur varieties, which presently account for about 15 and 10 per cent, respectively, of production.

University of California Farm Advisors, the California Avocado Society, and Calavo have been active in variety improvement programs. Calavo, in addition to disseminating information, has influenced growers' choices by its pooling procedures, by its classification of the varieties eligible for each grade, and by its announced unwillingness to handle many undesirable varieties after 1963. (See Calavo News, 1959.)

From the beginning, Fuertes have been unequaled in marketability. For comparison, table 2 indicates what percentage of the price per pound for Fuertes has typically been received by Calavo for avocados of other varieties (but of the same grade, size, and date as the Fuertes). The percentages
represent an average of the relative list prices that prevailed during a sample of weeks in about five postwar years. Comparative wholesale price and also a variety's historical importance are indicated for all varieties that represented at least 1 per cent of Calavo's deliveries during 1958-1959.

Qualities. Calavo employs a grading system based on a nine-category classification of varieties. Varieties are grouped so that selling prices will differ little within categories as compared with price differences between categories. Allowance is made, however, for the relative tonnage of varieties. Fuerte and Hass, the most important varieties, each form a separate category.

Table 2
LEADING CALIFORNIA VARIETIES: COMPARATIVE PRODUCTION AND PRICES

| Variety | Per cent of Calavo deliveries |  |  |  |  | Typical relative wholesale price |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1924 | 1930-31 | 1940-41 | 1950-51 | 1958-59 |  |
|  |  |  |  |  |  | per cent |
| Fuerte. | 10.1 | 46.0 | 67.3 | 79.4 | 61.0 | 100 |
| Hass. | .... | $\ldots$ | .... | 1.2 | 13.3 | 88 |
| MacArthur. | .... | $\ldots$ | 0.1 | 1.0 | 9.9 | 77 |
| Zutano. | .... | $\ldots$ | .... | 0.1 | 3.2 | 73 |
| Rincon. | $\ldots$. | .... | ... | .... | 2.5 | 90 |
| Nabal. | .... | ... | 12.2 | 4.1 | 2.0 | 65 |
| Anaheim. | ... | 3.4 | 3.9 | 3.2 | 1.7 | 48 |
| Dickenson. | 2.9 | 4.2 | 3.6 | 2.4 | 1.3 | 34 |
| 150 others. . | $\ldots$ | $\cdots$ | .... | $\ldots$ | 5.1 | 80 |

Source: Based on records of Calavo Growers of California.

Distinctions vary among the nine categories. For the Fuerte, Hass, and two other categories-representing altogether the 10 most palatable vari-eties-three qualities are regularly distinguished (plus up to four irregular qualities-for seedless, offbloom, "rusty," and weather-damaged fruit). The highest quality fruits of these varieties are grade-stamped "Calavo"; medium quality fruits are marked "Circle C"; low quality fruits are called "Standard" and are sold unsized.

For three other categories, two qualities are distinguished. High quality fruits of one of the categories are designated "Mexavo" and of the other two, "El Dorado." All lower quality fruits in the three categories receive the same "Standard" label that is applied to low quality avocados in the first four categories. Finally, the "Standard" label is also applied, without distinction, to all fruits in the two remaining categories.

Grading has been quite uniform since the "Calavo" grade was introduced in 1926, except for changes in the varieties eligible for that grade. The proportion of fruit graded "Calavo" or irregular equivalent has usually been about 60 per cent-although it has ranged from 49 (1951-1952) to 72 per cent (1934-1935). Table 3 indicates what percentage of the price per pound for Calavo grade avocados has typically been received by Calavo for avo-
cados of other grades (but of the same variety, size, and date as the Calavos). The percentages represent an average of the relative list prices that prevailed during a sample of weeks in about five postwar years. Also indicated is the historical composition of Calavo's sales according to grade.

The magnitude of the price differentials among different varieties and grades suggests that it would be appropriate to seek, not a single demand function for California avocados, but one for each variety-grade combination. On the other hand, both differentials have, over the years, been rather stable in percentage terms. This suggests that a single function, which of course is much simpler to obtain and to utilize, may suffice-especially if it contains variables representing the varietal and grade composition of sales. Indexes calculated for this purpose are presented in table 4 by seasons

Table 3
CALAVO GRADES: PROPORTIONS AND RELATIVE PRICES

| Grade | Per cent of Calavo deliveries |  |  |  |  | Typical relative wholesale price |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1926 | 1930-31 | 1940-41 | 1950-51 | 1958-59 |  |
|  |  |  |  |  |  | per cent |
| "Calavo'". | 57.1 | 59.8 | 62.8 | 59.2 | 66.6 | 100 |
| "Circle C". | .... | 13.9 | 19.5 | 18.7 | 20.3 | 84 |
| "El Dorado"' and "Mexavo". | . . . | 21.8 | 14.2 | 15.1 | 8.0 | 76 |
| "Standard"... |  | 4.5 | 3.5 | 7.0 | 5.1 | 35 |

Source: Based on records of Calavo Growers of California.
and in Appendix tables 4 and 5 (pp. 774-775) by months. The variety (or grade) index represents a weighted average: the sum over all varieties (or grades) of the proportion that each was of total deliveries, times the aforementioned relative value of that variety (or grade). Thus, the variety index would equal 100 if only Fuertes were sold, and the grade index would equal 100 if all sales were of Calavo grade.

Sizes. Market value is also related to fruit size. In general, with the relative supplies that have prevailed, and despite some channeling of larger fruit to eastern markets, Calavo sales records show that the smaller the fruit, the greater has been the price per pound. A recent study of consumer preferences, in Philadelphia, for Florida avocados "indicates that growers might expect to sell equal volumes of various sizes as long as they are priced on a weight basis" (Brooke, 1959, p. 6). For purposes of demand analysis, however, explicit account of size distribution seems unnecessary. The size distribution of the crop has been highly correlated with its varietal and grade composition and therefore is reflected in our variety and grade indexes.

In terms of orderly marketing, on-tree storage of mature avocados involves increases in fruit size and sometimes also losses in appearance and taste, as well as rotting and weather damage (plus theft in former years), but it also results in gains in weight and oil content. No satisfactory data
are available to measure these changes, but apparently, within limits, the weight gains from on-tree storage will more than offset the disadvantages (Freistadt, 1958, p. 39).

## Storeability

Avocados provide unusual opportunity for intraseasonal adjustment of supply. Such adjustment is made by storage of mature fruit, either on the trees or in the warehouse. The more important is storage on the trees, within

Table 4
INDEXES OF VARIETAL AND GRADE COMPOSITION
OF CALAVO SALES BY SEASONS, 1929-1959

|  | Variety index | Grade index |
| :---: | :---: | :---: |
|  | per cent | per cent |
| 1928-29 | 88.0 | 91.2 |
| 1929-30. | 81.3 | 87.4 |
| 1930-31. | 87.6 | 89.5 |
| 1931-32. | 89.7 | 89.5 |
| 1932-33. | 85.6 | 89.6 |
| 1933-34. | 83.6 | 87.1 |
| 1934-35. | 92.8 | 92.3 |
| 1935-36. | 87.7 | 90.5 |
| 1936-37. | 96.4 | 89.6 |
| 1937-38. | 95.1 | 90.2 |
| 1938-39. | 92.5 | 90.5 |
| 1939-40. | 94.0 | 93.3 |
| 1940-41. | 88.5 | 90.8 |
| 1941-42. | 90.7 | 91.5 |
| 1942-43. | 93.8 | 89.0 |
| 1943-44. | 93.7 | 90.3 |
| 1944-45. | 87.5 | 87.6 |
| 1945-46. | 94.4 | 90.6 |
| 1946-47. | 92.9 | 89.0 |
| 1947-48. | 90.7 | 89.5 |
| 1948-49. | 94.4 | 90.1 |
| 1949-50. | 93.6 | 88.6 |
| 1950-51. | 93.6 | 88.9 |
| 1951-52. | 95.1 | 86.0 |
| 1952-53. | 92.6 | 88.1 |
| 1953-54. | 93.0 | 89.4 |
| 1954-55. | 95.0 | 90.8 |
| 1955-56. | 90.0 | 90.5 |
| 1956-57. | 88.7 | 89.5 |
| 1957-58. | 94.0 | 92.7 |
| 1958-59. | 91.6 | 91.6 |

Source: Based on records of Calavo Growers of California.
limits set by seed sprouting, excessive oil, and other factors previously mentioned. Avocados do not start to ripen (soften) until they are picked, and they can be held on the trees longer than almost any other fruit. With warehouse storage, ripening is retarded by 40-degree temperatures. Avocados should be eaten one to four weeks after they are picked, depending on storage temperatures and on oil content. (The longer the fruit has remained on the tree, the shorter will be the period of satisfactory storage after pick-
ing, because oil content will be greater.) Together, the two kinds of storage allow Calavo to carry stocks of California avocados as long as 22 weeks beyond maturity (table 5).

## California Maturities

An avocado "is horticulturally mature when it softens and becomes edible on picking. ${ }^{\prime 5}$ In practice, criteria are variable and arbitrary. Criteria under the marketing order for Florida fruit relate to dates, colors, weights, sizes, and seed-coat color. In California, the only reliable criterion is generally considered to be oil content in a sample of fruits from the orchard-in particular, a minimum of approximately 8 per cent for Fuerte and most

Table 5
STOREABILITY OF CALIFORNIA AVOCADOS

| Maximum holding period without significant deterioration |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Type of storage | Fuerte | Hass | MacArthur | All others |
| On tree (barring unusual weather): Weeks, maturity to picking.. | 12-15 | 12-20 | 4-8 | 4-8 |
| In warehouse: |  |  |  |  |
| Weeks, picking to shipping: |  |  |  |  |
| For Los Angeles consumption. | 1-2 | 1-2 | 1-2 | 1-2 |
| For New York consumption. | 0.5 | 0.5 | 0.5 | 0.5 |

Source: Interviews.
other California varieties. In 1925 this criterion became part of a state law which forbids the picking of avocados with an oil content of less than 8 per cent (Senate of the State of California, 1958). Certain Florida handlers currently are challenging the validity of the law, since it also forbids the sale in California of any such avocados. Table 6 presents typical maturity patterns under this standard. However, maturities vary from year to year, and prediction is difficult. Furthermore, it should be noted that an oil content of 12 to 15 per cent, depending on variety, has been specified by Calavo Growers as minimum for Calavo grade. Consequently, for present purposes, availability is exaggerated by several weeks in table 6.

## Intraseasonal Supply Patterns

Because of storage possibilities, maturities set only upper limits on supplies. In the case of Cuban supplies, there was another limiting factor. Under a 1934 reciprocal trade agreement, shipments to the United States could clear a Cuban port only from June 1 through September 30 (a period during which most Cuban avocados normally mature). Imports from Cuba peaked about half the time in July and half in August.

[^2]The Florida season is much longer; shipping begins about June, peaks in November, and ends about March. In California, diversity of varieties and growing conditions allow avocados to be sold, as the maturity schedule suggests, throughout the year; sales invariably peak, however, around April, the heart of Fuerte harvesting.

Figure 2 shows estimated shipments from each source for the middle week of each month of 1954-1955 and 1955-1956, the most recent representative seasons. Figures for California represent the product of the annual

Table 6
MATURITY PATTERN OF CALIFORNIA AVOCADOS


Source: Interviews.
data given in table 1, times weekly percentages representing the sales pattern of Calavo Growers; shipments by independents are believed to be similar. Figures for Florida represent the product of the U. S. Dept. of Agriculture annual data cited with table 1 times weekly percentages calculated from reports of the Avocado Administrative Committee. Figures for imports represent the product of the U. S. Dept. of Agriculture annual data cited with table 1, times weekly percentages which were estimated from the monthly data on imports published by the U. S. Bureau of the Census. It is apparent that the predominance of California supplies suggested by annual data applies only to the winter and spring months. Cuban supplies were large in the summer months, and Florida avocados predominated in the fall.

Figure 2 also shows weekly weighted average selling prices, f.o.b. packinghouse, received by Calavo Growers for California avocados. Visual inspection gives no clear impression as to whether Calavo's prices are associated primarily with California or with total sales.


## Harvest Scheduling

Calavo exercises substantial influence on the intraseasonal distribution of its members' avocados. All members subscribe to a marketing contract which obliges them to deliver all of their commercial production to the association. Warehouse storage then occurs entirely at Calavo's discretion. Previous on-tree storage, although occurring while the fruit is still in members' possession, is influenced by helping to determine time of harvest. Indeed, the marketing contract confers on Calavo virtually complete control over harvesting decisions (Calavo Growers of California, 1953).
Division heads schedule the flow of fruit by specifying weekly tonnage targets. These are allotted geographically by the Field Department, and ultimately assigned to specific orchards and days by field representatives. Actually, in practice, the wishes of members with strong opinions as to the time their fruit should be harvested affect both the upward flow of information about fruit readiness and the downward flow of schedules.
The schedules are enforced in two ways: by availability of harvest labor, or by a penalty of approximately 2.5 cents per pound on unscheduled deliveries. Calavo both employs and arranges for picking crews, and many members prefer to pay the association to conduct their harvesting.
Nevertheless, once a delivery has been scheduled, no penalty is attached to variation from the expected tonnage. Because of such variation, daily deliveries do vary from the amounts scheduled. The aggregate variation for a day, however, is seldom more than 10 per cent. This is small enough, relative to incoming and outgoing inventories, not to disturb packing or selling operations.

## Predictability of Production

Intraseasonal adjustment of supplies must be made in the face of considerable uncertainty not only about maturities and the yields of specific picks, but also about total production. A study of Calavo's experience, over several seasons, in forecasting its members' deliveries suggests two pieces of information that will be needed later.
First, if we suppose that the figure that ultimately proves correct for a season's harvest is (over the years) distributed virtually normally around a point estimate made at the beginning of the season, the standard deviation will be about 10 per cent of the initial estimate. That is, about twothirds of the true values will differ from the initial estimates by less than 10 per cent of the latter. (One bit of evidence suggested a rather smaller standard deviation. This was the answer that Calavo officials gave in October, 1959, when asked to name, for the season then beginning, a figure that was "almost certain" to be exceeded and another "almost certain" not to be exceeded. Each differed from the point estimate by only 10 per cent. Other information, however, suggested a rather larger standard deviation. For 1957-1958 and 1958-1959, the true values proved to be 115 and 118
per cent, respectively, of rough initial estimates; and for 1959-1960 the true value was 91 per cent of a careful estimate.)

Second, whether the initial estimate proves too low or too high and whether the difference proves large or small, if a new estimate is made after $x$ per cent of the true total crop has been harvested, the absolute error in the new estimate will prove to be about $(100-x)$ per cent of the error in the initial estimate and of the same sign. That is, the error in forecasts will decline in proportion to the percentage of the crop already delivered.

Strictly, this formula implies that an initial estimate that turns out to be, for example, 10 per cent of the true value too high, represents a sum of forecasted monthly deliveries each of which is also 10 per cent too high and none of which are revised in the light of previous errors when new estimates are made. Nevertheless, as an approximation to actual experience, the formula is a rather good fit. Thus, an estimate of 1959-1960 deliveries that was made in June, 1960, a time at which 75.4 per cent of the crop had been delivered, proved to be 1.2 million pounds ( 2.1 per cent) too high. In comparison, multiplying the initial overestimate ( 5.7 million pounds) times ( 100 -75.4 ) gives 1.4 million pounds ( 2.4 per cent) too high. For a forecast made in May, 1959, the formula implied an underestimate of 2.1 million pounds (4.5 per cent). The actual error was an underestimate of 1.1 million pounds (2.4 per cent).

The errors in these forecasts are attributable primarily to variability in yields. Calavo's predictions as to future seasons are further complicated by uncertainty as to the addition or termination of members, and also by variability in the total number of bearing trees.

## Supply Response Velocities

Supply reduction can, of course, occur quite rapidly. In the extreme, the crop might not be harvested if expected returns would fall below picking and hauling costs. To date this has not happened for commercial-size growers. Expansion is a different story.

California avocados mature, depending on the variety, seven to 18 months after blooming. An increase in (expected) prices could influence production during the following year by inducing more intensive orchard management or harvesting. Given current practices, the influence apparently would be slight. Some increase in production might occur in two years. In two years a good crop can be obtained from avocado trees top-worked to betteryielding or differently-maturing varieties. For a substantial increase (or relocation) of production, however, additional trees must bear; and with orchard development, it is likely to be at least five years before even moderate yields are safely available. But orchard development in turn presupposes the availability of nursery trees, and about 30 months elapse from the time the seed is planted until the budded or grafted trees are ready to plant (Hodgson, 1947, p. 20). In sum, only long-run types of adjustment would substantially increase production, and substantial increases would occur only after five to
eight years from the point of decision (not stimulus). The continuing expansion of bearing acreage is a response to events long past.

If we were to fit an annual supply function, therefore, it would contain substantial lags. For present purposes, however, these lags, and the overwhelming importance of variations in yields, suggest that we shall not be very wrong in supposing that annual harvests are independent of current prices-at least for prices above harvest costs. Harvest costs in California may average 1 to 2 cents per pound. (Total costs are at least 6 cents and average perhaps 10 to 15 cents, of which about 30 per cent represents depreciation and interest. Irrigation costs in San Diego County represent about 30 per cent of total costs. (See California Agricultural Extension Service, 1953, 1954, 1956.) Mature trees in Florida seldom need irrigation (Ruehle, 1958, p. 5). Information on comparative labor costs has been presented by Gavet (1958).)

The length of the delay in supply response also suggests that the problem of higher returns attracting additional competitive production may be disregarded. This implies approaching the problem of orderly marketing with only short-run considerations in mind.

## Handling Costs

If harvesting is to be remunerative to Calavo members, average selling price must exceed not only harvest costs but also Calavo's packing and other variable costs. Calavo owns two packinghouses, 150 miles apart, and also has had packing done from January to September at an orange packinghouse about halfway between the other two. The maximum rate of output for the three together at designed conveyor belt speeds is 400,000 pounds per eight-hour day, or two million pounds per standard week. Additional output can be achieved by lengthening both the work day and the work week, but overtime pay rates are substantial, and limited cooler space creates a bottleneck. Tonnage actually packed in the last decade has ranged from 40 to 2,000 thousand pounds per week and from 17.1 to 54.4 million pounds per season.

Data are not available from which reliable estimates could be made of the actual or the potential relation between Calavo's costs per pound, f.o.b. packinghouse, and the number of pounds sold per period. A crude estimate has been made on the basis of data that are available. These include figures computed each season for Calavo's "packing, field, and other production costs of California avocados," "other marketing expenses," including metropolitan delivery, and "general office expense,"-that is, total operating costs, including depreciation, except those falling under "advertising and promotion" and "transportation of California avocados" (Calavo Growers of California, annual reports).

By means of these data, the following estimate was obtained of Calavo's unit handling costs for a range of, for example, 0.2 to 2.0 million pounds per week: $u=3.51 q^{-1}+3.0-0.5 q$, where $u$ represents cost per pound, in

cents, and $q$ represents million pounds sold per week. ${ }^{6}$ Marginal handling costs then are estimated as $c^{\prime}=3.0-1.0 q .{ }^{7}$ Both functions are shown in figure 3, along with the meager data on which they are based-four annual observations. ${ }^{8}$

It is apparent that Calavo's f.o.b. costs per pound have fallen considerably when tonnage sold has increased. Selling prices, however, have fallen even more, so that the percentage of sales that these costs represent has increased in years of large crops. In contrast, expenses falling in the category "advertising and promotion" show no consistent relation with either volume or sales. The figures have varied more than 100 per cent from year to year, both per pound and per dollar of sales, fluctuating with the annual budget. Intercity transportation costs, on the other hand, have been quite steady from year to year on a per-pound basis (although they have drifted upward secularly). Calavo's eastern shipments usually move by rail; its shorter western shipments, by truck. Most Florida shipments move by truck. As a result, relative to Calavo's sales, transportation costs have increased markedly (to 10 per cent) in years of large crops. The over-all breakdown of consumers' dollar in recent years is indicated in table 7.

[^3]
## Selling Policies

Avocado growers sell their fruit (sometimes on the trees) at roadside, to local grocers, through commission houses, and to fruit buyers and packinghouses. From the packinghouses, avocados move to jobbers, wholesalers, integrated retailers, and institutions. In the case of the smaller packers, sales may be made at the packinghouse, by telephone, at a nearby produce market, or through brokers and commission houses. Terms of sale are subject to bargaining. In sales of California avocados at both the grower and jobber levels, however, Calavo's prices set a standard that independent handlers must attempt to exceed in buying and undercut in selling.

Table 7
DIVISION OF CONSUMERS' OUTLAYS, 1955-1959

| Factor | Weighted averages, 1955-56-1958-59 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Per pound |  | Per dollar |  |
| Distributors.. | cents | $\begin{gathered} \text { cents } \\ 14 \end{gathered}$ | per cent | $\begin{gathered} \text { per cent } \\ 40 \end{gathered}$ |
| Calavo: |  |  |  |  |
| Packing......... | 4.0 | . | 11.3 | . |
| Office*.. | 2.5 | . | 7.1 | . |
| Promotion...... | 0.5 |  | 1.4 | $\cdots$ |
| Transportation.. | 1.5 | 8.5 | 4.2 | 24 |
| Member-Growers. . |  | 12.5 |  | 36 |
| Consumer cost... |  | 35 |  | 100 |

[^4]From the beginning, and on all transactions, Calavo's selling practices have involved the naming of list prices, f.o.b. Los Angeles or packinghouse. "Asking prices" are quoted for about 90 different variety-grade-size categories of fruit over the course of a season. Predetermined zone differentials are added if intercity transportation is involved, along with a warehousing charge to branch office customers who want delivery immediately instead of from the packinghouse. About one sale in four by the branch offices, however, is f.o.b. packinghouse. The zone differentials are somewhat arbitrary, introducing a subtle form of price differentiation among the quotations of different sales offices. With respect to California versus outside net prices, it is important to Calavo that the former not be higher, since nonmembers' fruit is sold proportionately more inside the state. In any case, many larger customers prefer to arrange their own shipments from California, often mixing commodities. Volume discounts, based on monthly purchases, are also allowed.

The price list is revised frequently, customers who patronize only Calavo being reimbursed for inventory losses when price reductions occur. Prices
are revised in accordance with trends in packinghouse inventories and in off-list sales. As the latter implies, the list is viewed not as an expectation but as a target.

Given the market situation pictured above, Calavo clearly has an opportunity to affect its net returns by adjusting its marketings-provided information can be obtained about demand.

## DEMAND FOR CALAVO AVOCADOS

Two kinds of functions relating price paid to quantity purchased must be distinguished.

One is ex ante demand-a hypothetical relation that summarizes buyers' preferences or intentions. The most useful concept of ex ante demand is that it expresses, for particular values of income and other shift variables (including previous periods' prices), the quantities that would be purchased during a period if various alternative single prices (or sets of single prices, if different grades, etc., are involved) were offered. ${ }^{9}$

The other function is obtainable. It is ex post demand-an historical relation obtained by analysis of time series. It expresses the average of the average prices that historically were associated with various quantities sold, after allowing for the effect of changes in various shift variables. Alternatively, it may express the average of the quantities that were associated with various average prices.

Ex ante and ex post demands do not necessarily converge as statistical problems are resolved. In general they will differ if prices have varied within the time periods considered.

A difference arises because the average price that is actually obtained for a given quantity depends on how individual prices vary; average price is lower if single prices prevail than if buyers suffer discrimination. Conversely, the quantity that is sold at a given average price is less if prices are all equal than if some intramarginal buyers pay more than the $e x$ ante price and some extramarginal buyers pay less. Such intraperiod price variation may result from price concessions to large or reluctant buyers, from geographic discrimination, from discrimination among subperiods, from a pattern of price variation inherent in the transaction system, ${ }^{10}$ or from other conse-

[^5]quences of prevailing marketing practices. In any event, when price variation has occurred systematically, the average quantities historically associated with various average prices will differ from the ex ante demand that prevailed. As a result, the ex post function will not reproduce the ex ante function. Instead, the parameters of the ex post function will incorporate the effects of the historic pattern of intraperiod price variation.

In the present study this discrepancy creates no problems because the study is oriented toward the effects of intraseasonal quantity reallocations on seasonal income under the prevailing market structure and conduct. What is needed is the weekly average price that Calavo actually would receive for each alternative weekly quantity under prevailing marketing practices, whether or not those practices cause the average location of ex post average price to lie off an ex ante function. This actual value is exactly what an ex post function indicates.
On the other hand, the very process of reallocating weekly quantities would alter the relation between annual average price and annual quantity that a future analysis of annual data would disclose. Conversely, the ex post demand obtained by analysis of past annual data must be understood to reflect the specific pattern of intraseasonal allocation that has prevailed during the period studied. The annual demand obtained is merely one of a family of ex post relations that might have held with the given ex ante demand. This, too, is no problem, since the function obtained is not used for the intraseasonal solution.

On the other hand, important statistical deficiencies remain. Arbitrariness or error enters in various ways: (1) in assuming that variables are predetermined; (2) in specifying a form of the function; (3) in using a single form for extended time spans; (4) in neglecting other explanatory variables and lags; (5) in specifying a criterion of best fit; (6) in appraising statistical significance; (7) in rejecting results when parameters have objectionable signs or relative magnitudes; (8) in using erroneous data; (9) in inferring population parameters from sample parameters; (10) in presuming that past relations still hold; and (11) in extrapolating for purposes of prediction.
Despite these deficiencies, a useful approximation may be obtained by fitting a single least-squares multiple-regression equation with price the endogenous variable, and regarding a period's fresh sales (and other variables) as substantially unaffected by current price. The latter is a plausible assumption for the two time periods to be considered. It is plausible for annual data because production is annual and nature-dependent, harvests are virtually equal to fresh sales, use-value to growers is slight, carry-over at year's end is relatively small, and imports have a level trend. At the other extreme, a week is short enough so that harvest schedules may have been substantially predetermined and variations in warehouse inventories may have been substantially restricted by perishability.

## Annual Demand

Final Equation. The final estimate, based on data for 1939-1940 through 1958-1959 (see table 1 and Appendix table 8), is:

$$
\left.\begin{array}{r}
\hat{P}=-66.1-0.327 C+44.6 \log Y,\left\{\begin{array}{l}
R^{2}=0.827 \\
(t=7.37)(t=8.68)
\end{array}\right\}_{1.23}=3.4
\end{array}\right\}^{11}
$$

where $\hat{P}=$ "predicted" season-average Calavo selling price, f.o.b. Los Angeles, in cents per pound;
$C=$ million pounds of California avocados sold during the season; and
$Y=$ billion dollars of United States nonagricultural personal income (simple average of 12 seasonally adjusted monthly figures).

That is, for each additional million California pounds sold per year, Calavo's actual (not list) selling prices decreased an average of 0.327 cent per pound; for each 5.3 per cent increase in nonagricultural income, prices increased an average of 1 cent per pound. ${ }^{12}$ Of the total variance in annual average prices, 83 per cent could be represented as the (specified kinds of) effect of concurrent changes in volume or income. Prices for the period, if estimated from the equation, would have been less than 3.4 cents wrong for about two-thirds of the years.

How responsive price was to changes in volume and income is also indicated by coefficients of price flexibility. The coefficient of price flexibility with respect to volume was -0.69 at the centroid. That is, at the average level of volume ( 44.9 million pounds) and price ( 21.2 cents), a 1 per cent increase in volume implied a 0.69 per cent reduction in price-and therefore greater gross income. The coefficient of price flexibility with respect to income was 0.91 at the centroid. That is, at the average level of price, a 1 per cent increase in income was associated with a 0.91 per cent increase in price. ${ }^{13}$

[^6]The implied net relation between Calavo price and Calavo volume $(Q)$ for the 1958-1959 level of nonagricultural income and of other California sales is shown by the line in figure 4 labeled "Predicted." The intercept of this line was obtained by calculating the price ( $\hat{P}_{59}$ ) that the annual demand equation predicts for the 1958-1959 level of income and of total California sales, and adding the increase in 1958-1959 price $\left(0.327 Q_{59}\right)$ that the equation indicates would have occurred if Calavo volume ( $Q_{59}$ ) had been zero. The vertical distances between this line and the dots show how accurately the original equation "predicted" prices at each observed level of Calavo volume. That is, the vertical distance above a Calavo volume represents the difference between the historic price associated with it and the price the original equation predicts when given that Calavo volume and the income and the other California volume prevailing in the year with that Calavo volume. (The residual for 1958-1959 happened to be zero.) The curve labeled "maximum attainable" shows the season-average prices that might have been attained in 1958-1959 if the intraseasonal allocation of each annual volume was optimal; the derivation of this curve is explained on page 744.
Methods. Numerous impressionistic judgments were required. These related primarily to the time period used, to the variables tried, to the variables deleted and retained, and to the form of the function.

The Time Period. Regressions were obtained for four time spans, differing in starting points. The first started with 1924-1925, the earliest season for which annual data were available. Regressions for this period contained sizeable negative time trends, because Calavo's prices in the 1920's were about double any occurring thereafter. No similar trend appeared for later periods, and in any case $R$ was not high. The second period started with 1930-1931. The coefficients obtained here were not significantly different from those of the third period, but $R$ was somewhat lower. The third period, starting with 1939-1940 (when the author's weekly series began), eventually was adopted. It includes the war years, but no controls were imposed on avocados, and the data and residuals suggest that essential continuity existed. In any event, the war years were omitted in the fourth period, which started with 1945-1946. In both the annual and weekly analyses the coefficient of quantity was not significantly altered, and $R$ was usually improved. The coefficient of income, however, was appreciably smaller and, it was felt, gave a less reliable account of the effect of sizeable changes in money incomes

[^7]

Fig. 4. Annual Calavo demand in 1958-1959, predicted (with residuals), and maximum attainable.
-as indicated by the presence of a large upward bias when the equation was used to "predict" prices during the prewar period.
Variables Deleted. Five variables did not pass tests of significance. The coefficient of year at times was positive and at times negative, but rarely large compared with its standard error. Because year and income are highly correlated for recent periods, the effect of any evolutionary changes (for example, in demographic characteristics) may be reflected in the final income coefficient. Mention was made earlier of a variety index and a grade index. Their coefficients, perversely, usually were negative, but in any event not significant at any acceptable level.

Florida sales and imports were tried both individually and summed. The coefficients obtained were usually negative, as would be expected, but fluctuated widely and sometimes were even (absolutely) larger than the California coefficient. At times the coefficients were quite significant, but always in equations that were otherwise objectionable. When added to the final equation shown above, none could qualify at even a 10 per cent level of significance. The model used may be inappropriate, or it may be that Florida and imported volume did not appreciably affect annual California average price because of their seasonality, relative magnitude, and preponderantly eastern distribution.

Variables Retained. That price should be the dependent variable follows from using the single-equation approach. Price to Calavo was chosen because of the availability of reliable data and the orientation of the study. In contrast, combining Calavo with other California fresh sales was a last resort. In both the annual and weekly analyses, when independents' volume was treated separately, the coefficient was seldom significant, and often positive. This occurred even when either or both California quantities were expressed in logarithms and even when (in the weekly analysis) all variables were expressed as first differences. (The coefficient of correlation between Calavo and other California annual volume was 0.926 ; for the (inferior) weekly data, it was 0.906 .) Forcing the independents' coefficient to equality with Calavo's seemed more reasonable than accepting it to be either positive or zero.

The resulting relation of Calavo price to total California volume emerged linear. Linearity must be regarded as an approximation, since it is implausible that the same reduction in price would be associated with an increase in volume from 20 to 21 million pounds as with an increase from 90 to 91 million pounds. Linearity may, however, be a fair approximation. Use of $\log C$ instead of $C$ did not yield appreciably better results. Furthermore, while the scatter of residuals from the final linear arithmetic relation shown above was somewhat objectionable, the relation could be assumed reasonably accurate for the range covered according to Foote's (1958, p. 174) test for nonlinearity.

Nonagricultural income was employed for reasons mentioned earlier, despite the fact that it is not net of taxes. It was tried in both arithmetic and logarithmic forms. The latter usually gave a somewhat better result, in terms of both $t$ and $R$. That the second partial with respect to income itself should be negative is plausible, since changes in price levels were not introduced explicitly. (For present purposes it appeared preferable to avoid the complications and ambiguities that would result from dividing other variables by, or adding a variable for, either a price level index or population.) If demand is proportional to price level, and increases at a decreasing rate with real income, the additive relation would be curvilinear. The Foote (1958) test for nonlinearity applied to the relation between price and $\log Y$ proved inconclusive. (Because the array of residuals by year and the array
by income were identical, the Foote test and the Durbin-Watson test for serial correlation were identical. The latter, therefore, also was inconclusive.) However, the scatter of residuals indicated that $\partial^{2} \hat{P} / \partial Y^{2}$ is, not zero, but even more negative.

## Weekly Demand

A weekly demand equation may for various reasons omit important shift variables which vary systematically over the course of a season. Then different equations should be shown for different weeks of the season. Three techniques have been used which allow for intraseasonal shifts.

Alternative Approaches to Intraseasonal Shifting. The first distinguishes subperiods (e.g., weeks, or summer-winter) and treats the data for each subperiod over a number of years as a separate set of observations (Mehren and Erdman, 1946; Hoos and Seltzer, 1952; Hoos and Boles, 1953). Regressions are obtained for each set, generally of the same form. Differences in the results may then be appraised by significance tests or by impressions about patterns. Impressions may come, for example, from comparing parameters listed in sequence, or from seeing whether a graph of the functions displays an "orderly fan-shaped arrangement" (Mehren and Erdman, 1946, p. 594).

The second approach treats all observations as a single set and obtains a single regression equation. Each observation, however, is quantified in $m-1$ additional dimensions, where $m$ is the number of subperiods distinguished. Each of these dimensions is assigned a value of unity for all observations belonging to an associated subperiod, and a value of zero for all other subperiods. The regression equation then contains $m-1$ dummy variables. The coefficients of these variables provide for arithmetic (or, with a power function, proportionate) intraseasonal shifts in the height of each subperiod's demand relative to the height in the $m$-th subperiod. The presence of intraseasonal shifts is appraised by the statistical significance, and perhaps the orderliness, of these coefficients.

The third approach also treats all observations as a single set and obtains a single regression equation. This time, however, each observation is quantified in one additional dimension. It represents the "value" of the associated subperiod-for example, the number (1-52) of the week in which the result occurred. "Week," then, becomes an additional variable in the regression equation, introduced additively, in product terms, etc. (Foytik, 1951). The presence of intraseasonal shifts is appraised by the statistical significance of coefficients of the terms that include "week."

The first and third approaches have been employed here. The first, or "independent functions" approach, permitted intraseasonal patterns to become manifest with relatively little guesswork and (unlike the second approach) with relatively few restrictions on the character of intraseasonal shifts. The third, or "generalized function" approach, permitted the apparent patterns to be systematized, related to a larger number of observations, and interpolated from the weeks sampled to all weeks of the season.

Table 8
REGRESSION EQUATIONS FOR INDEPENDENT WEEKLY DEMAND FUNCTIONS (1940-1958)

| Week of the season | Average ending date | $\begin{aligned} & \text { Constant } \\ & \text { term } \end{aligned}$ | Net regression coefficients (with tratios) |  |  |  | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $c$ | $\log y$ | $f$ | $i$ |  |
| 3. | 10/20 | -80.3 | $\begin{array}{r} -38.2 \\ (5.36) \end{array}$ | $\begin{aligned} & 51.5 \\ & (8.81) \end{aligned}$ | $\ldots$ | $\ldots$ | 0.829 |
| 7. | 11/17 | -64.4 | $\begin{array}{r} -33.9 \\ (3.54) \end{array}$ | $\begin{aligned} & 45.6 \\ & (8.76) \end{aligned}$ | $\ldots$ | $\ldots$ | 0.618 |
| 12. | 12/22 | -50.6 | $\begin{gathered} -13.9 \\ (1.90) \end{gathered}$ | 36.4 $(6.32)$ | $\ldots$ | $\ldots$ | 0.555 |
| 16. | 1/19 | -50.8 | $\begin{array}{r} -12.7 \\ (2.96) \end{array}$ | $\begin{aligned} & 37.1 \\ & (9.69) \end{aligned}$ | $\ldots$ | $\ldots$ | 0.598 |
| 20. | 2/16 | -57.1 | $\begin{gathered} -10.1 \\ (4.81) \end{gathered}$ | $\begin{gathered} 39.7 \\ (11.92) \end{gathered}$ | $\ldots$ | $\ldots$ | 0.700 |
| 24. | 3/16 | -68.2 | $\begin{gathered} -10.8 \\ (5.98) \end{gathered}$ | $\begin{gathered} 46.3 \\ (13.77) \end{gathered}$ | $\ldots$ | $\ldots$ | 0.796 |
| 29. | 4/20 | -65.2 | $\begin{array}{r} -11.8 \\ (6.03) \end{array}$ | $\begin{gathered} 45.8 \\ (15.84) \end{gathered}$ | $\ldots$ | $\ldots$ | 0.792 |
| 33. | 5/18 | -69.4 | $\begin{gathered} -10.9 \\ (6.50) \end{gathered}$ | $\begin{gathered} 46.6 \\ (11.12) \end{gathered}$ | $\ldots$ | $\ldots$ | 0.801 |
| 38. | 6/22 | -64.0 | $\begin{gathered} -15.3 \\ (6.43) \end{gathered}$ | $\begin{gathered} 43.8 \\ (18.52) \end{gathered}$ | $\ldots$ | $\cdots$ | 0.798 |
| 42. | 7/20 | -59.2 | $\begin{array}{\|r\|} -18.6 \\ (6.84) \end{array}$ | $\begin{aligned} & 40.7 \\ & (7.92) \end{aligned}$ | $\cdots$ | $\begin{gathered} -5.72 \\ (2.00) \end{gathered}$ | 0.831 |
| 46. | 8/17 | -71.5 | $\begin{array}{\|c} -20.3 \\ (6.76) \end{array}$ | $\begin{aligned} & 45.2 \\ & (7.55) \end{aligned}$ | $\begin{aligned} & -15.4 \\ & (1.89) \end{aligned}$ | .... | 0.841 |
| 51... | 9/21 | -66.6 | $\begin{array}{\|} -25.1 \\ (4.50) \end{array}$ | $\begin{gathered} 43.3 \\ \quad(6.54) \end{gathered}$ | ... | $\ldots$ | $\frac{0.736}{0.742^{*}}$ |

* Average using total variances as weights.

Independent Functions. To limit the study to manageable proportions, equations were obtained for only one week in each month-a middle week, to facilitate use of data available only by months.

Final Equations. Table 8 presents the final estimates, based on data for 1939-1940 through 1957-1958 (see Appendix tables 1, 2, 6, 7, 8). (Data for 1958-1959 were not available at the time of calculation.) The dependent variable is "predicted" weekly average Calavo selling price, f.o.b. Los Angeles, in cents per pound, and:
$c=$ million pounds of California avocados sold during the week;
$y=$ seasonally adjusted annual rate of nonagricultural personal income for the week, in billion dollars;
$f=$ million pounds of Florida avocados sold during the week; and
$i=$ million pounds of imported avocados sold during the week.
Variables Deleted. Five variables did not pass tests of significance. Coefficients of year ran consistently negative for the eight weeks 7 to 38 , and inclusion of year substantially increased their coefficients of determination. However, only one coefficient was significant at the 5 per cent level, and only three at the 10 per cent level. These results seemed to warrant
trying "year" in the generalized function, but when it did not prove useful there, it was deleted here, too.

Coefficients of the variety and grade indexes (Appendix tables 4 and 5) tended to be positive but were rarely significant. In view of their nonsignificance in the annual analysis, too, they were not included in the generalized function.

Two variables not yet mentioned were also tried. One was the weekly mean of daily maximum temperatures at Los Angeles, the most important avocado market. Temperature appears to be an important shift variable in the demand for lemons (Hoos and Seltzer, 1952). Here, the $t$ ratios were almost invariably near zero. In view of these results, temperature was not tried in the generalized function.

The other variable was Calavo quantity during the previous week (e.g., during week 2 in the equation for week 3; see Appendix table 3). In the case of weekly auction demand for plums, price appears to be inversely related to both the current and the previous week's quantity (Foytik, 1951). The explanation may lie in a tendency for consumers to vary their diets. Also, carry-over may be important. The latter could be especially significant with avocados, since they may have over a week's shelf life both when the retailer and the housewife acquire them. In fact, negative coefficients usually were obtained, but they were not often significant. After trying and deleting lagged quantity in the generalized function, it was also excluded from the final equations here (and no attempt was made to test for other types of lags and leads).

Intraseasonal Shifts. The striking thing about the final equations shown in table 8 is the fact that the absolute value of the coefficient of quantity, with one exception, decreases until near midseason and then increases. In fact, the values (including the one for week 12, which is not significant at the 5 per cent level) closely approximate a simple parabola. This apparently systematic variation suggested a form for the generalized function.

In contrast, the constant term and the coefficient of income appear to vary not systematically but randomly-an impression later supported by generalized-function results.

The weekly series for Florida volume and imports may contain substantial errors. These may explain, in part, why no pattern of variation in their coefficients was discernible. At the 5 per cent level, their coefficients did not test significantly different from zero for even one week, and at the 10 per cent level, coefficients were significant for only one week each. For imports, the week was number 42, which falls in July, often a peak month. For Florida quantity, however, it was week 46-mid-August-which is appreciably before the peak of the Florida season. Even if both weeks were peak, however, it is implausible that Florida quantity and imports affect Calavo prices during only one week (or month) of the season. Nevertheless, the relative magnitudes of the coefficients are plausible, and served as a check on the generalized function.

The Generalized Function. When the 19 observations for each of the 12 selected weeks were treated as a single set of data, and figures for 19581959 (which were not originally available) were added, the sample contained 240 observations (see Appendix tables 1, 2, 6, 7, 8).

Final Equation. The final estimate was:

$$
\begin{gathered}
\hat{p}=-69.5-28.4 c+1.40 c w-0.0274 c w^{2}-7.14 n \\
(t=10.79)(t=7.76)(t=8.50)(t=7.26) \\
+46.7 \log y,\left\{\begin{array}{l}
R^{2}=0.748 \\
S_{1.2345}=4.4
\end{array}\right\},
\end{gathered}
$$

where $\hat{p}=$ "predicted" weekly average Calavo selling price, f.o.b. Los Angeles, in cents per pound;
$c=$ million pounds of California avocados sold during the week;
$w=$ week of the California season;
$n=$ million pounds of Florida and imported avocados sold during the week; and
$y=$ seasonally adjusted annual rate of nonagricultural personal income for the week, in billion dollars. (Sources of the income series are given in Appendix table 8, p. 776.)

The coefficient of determination indicates that 75 per cent of the total variance in weekly average prices could be accounted for as the indicated kind of effect of simultaneous changes in $c, n, y$, and $w$. This figure is somewhat less than the annual one ( 83 per cent). However, it compares quite favorably with the determination of the independent weekly functions. The weighted average for the latter, as indicated in table 8 , is 0.742 . For the generalized function, $R^{2}$ is 0.748 ; more relevant, $R^{2}$ was 0.732 when the parameters of the generalized function initially were obtained from the same 19 years' data that were available when the independent functions were obtained. ${ }^{14}$ That is, determination here, given the same data, is only 0.010 less than the figure for independent functions. This is impressive; the generalized function had less freedom to increase $R^{2}$ by irregular adjustments to the raw data since its form would not allow intraseasonal shifts to be unsystematic.

Intraseasonal Variation. The form of the generalized function implies that the constant term and the coefficients of both log-income and nonCalifornia volume are the same for all weeks, while the net California pricequantity relation varies systematically over a season. In contrast, Foytik (1951, p. 447) obtained intraseasonal shifts in both the constant term and the coefficient of income, and fixity in the quantity coefficient. He commented that "From a theoretical point of view such a shift [in the income coefficient] is justified."

[^8]Shift Variables. In earlier runs the constant term was permitted to vary during the season by introducing $w$ and $w^{2}$ as separate terms (as well as components of product terms). The $t$ ratios of both were close to zero. This was true even after all product terms other than those involving $c$ had been deleted, at which point the coefficients of $w$ and $w^{2}$ might have incorporated systematic intraseasonal changes in the influence of income, non-California volume, or other variables. (Since the income variable was seasonally adjusted, the effect of seasonality in actual income might have been incorporated even before the product terms in income had been deleted.) The resulting uniform constant is approximately an average of the scattered constants obtained by using independent weekly functions.
The product terms $w \log y$ and $w^{2} \log y$ were also deleted because of low $t$ ratios. The resulting fixed coefficient of $\log y$ in no case differs from the coefficients obtained by using independent functions by more than two standard errors of the latter. The coefficient here is almost the same as that in the annual analysis; here, for each 5.1 per cent increase in nonagricultural income, prices-now in weekly-average terms-increased an average of 1 cent per pound. The Foote (1958) test for nonlinearity, applied to residuals for weeks 12,29 , and 46 , each treated as a separate set, was once inconclusive and twice indicated nonlinearity in the relation between price and $\log y$. As with the annual residuals, however, scatter diagrams indicated the reason for this-the relation between price and income itself was not less, but more curvilinear. With the curve employed, the coefficient of price flexibility with respect to income was 0.92 at the weekly centroid.

The coefficient of $n$ indicates that for each additional 100,000 pounds of Florida or imported avocados sold per week, Calavo's selling prices decreased an average of 0.7 cent per pound. At first, Florida quantity and imports were treated as separate variables. However, the coefficient of imports alone, while highly significant, was almost double that of Florida quantity. This was an implausible result. In terms of physical characteristics and price, Florida avocados are the closer substitute. Furthermore, the difference in coefficients could not be associated with differences in the coefficient of California quantity; the value of the latter when imports peak is close to the value when Florida quantity peaks. The difference also contrasted sharply with the results obtained by use of independent weekly functions. Consequently, the hypothesis that the "true" Florida coefficient was equal to the "true" import coefficient was tested. When the hypothesis could not be rejected at even the 10 per cent level, it seemed appropriate to combine the two quantities. The resulting single coefficient does not differ from either coefficient obtained with independent functions by more than two standard errors of the latter.

Before Florida volume and imports were combined, product terms of each with $w$ and $w^{2}$ were tried. The $t$ ratios invariably were close to zero. In addition, the Foote (1958) test for nonlinearity was applied to two weeks' residuals from the final equation. For week 46, the test was inconclusive; for week 12 , linearity could be assumed.
Table 9

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Week of the season | $\begin{gathered} \text { Average } \\ \text { ending } \\ \text { date } \end{gathered}$ | ${ }^{w}$ w, 19:9 | $b_{w}$ | $\begin{gathered} \hat{p} \text { for } \bar{c}_{w}, \\ \bar{n}_{w,}, \operatorname{meman}_{w} \\ \log y \end{gathered}$ | Mean values: |  |  |  | $\begin{aligned} & \text { Price } \\ & \text { flexibility at } \\ & \text { centroid*** } \end{aligned}$ | $\underset{\substack{\text { Price } \\ \text { elasticity at } \\ \text { centroid } \dagger}}{ }$ |
|  |  |  |  |  | $\bar{p}$ | $\bar{c}$ | $\bar{n}$ | antilog of mean $\log y$ |  |  |
| 3. | 10/20 | 34.2 | -24.4 | 24.9 | 24.6 | 0.33 | 0.49 | 186 | ${ }^{-0.32}$ | -3.1 |
|  | 11/17 | 29.7 | -19.9 | 24.4 | 26.8 | 0.37 | 0.68 | 188 | $-0.30$ | -3.3 |
| 12 | 12/22 | 25.1 | -15.5 | 20.9 | 21.4 | 0.78 | 0.54 | 190 | $-0.58$ | $-1.7$ |
| 16. | 1/19 | 24.4 | -12.9 | 22.5 | 21.4 | 0.93 | 0.35 | 191 | $-0.53$ | -1.9 |
| 20 | 2/16 | 27.5 | -11.3 | 20.5 | 19.9 | 1.35 | 0.19 | 192 | -0.74 | -1.4 |
| 24. | 3/16 | 28.3 | -10.5 | 21.0 | 21.2 | 1.54 | 0.01 | 193 | -0.77 | -1.3 |
| 29. | 4/20 | 28.3 | -10.7 | 20.9 | 21.6 | 1.53 | 0 | 194 | -0.78 | $-1.3$ |
| 33. | 5/18 | 28.0 | -11.9 | 20.6 | 21.8 | 1.42 | 0 | 196 | -0.82 | -1.2 |
| 38. | 6/22 | 27.2 | -14.6 | 25.1 | 24.0 | 0.81 | 0.11 | 198 | -0.47 | -2.1 |
| 42. | 7/20 | 29.5 | -17.7 | 22.6 | 20.2 | 0.61 | 0.62 | 199 | -0.48 | -2.1 |
| 46. | 8/17 | 30.3 | -21.7 | 21.5 | 19.8 | 0.52 | 0.72 | 200 | $-0.52$ | -1.9 |
| 51. | 9/21 | 31.0 | -28.0 | 21.5 | 22.6 | 0.42 | 0.68 | 202 | -0.55 | -1.8 |
| All weeks. | $\ldots$ | $\ldots$ | . |  | 22.1 | 0.87 | 0.36 | 194 | ..... | $\ldots$. |

[^9]Price-quantity Relations. In contrast with the constant term and the coefficients of $\log y$ and $n$, the coefficient of California quantity is shown to vary during the season; that is, even if income and non-California volume hold constant, different prices are necessary to move a given quantity at different times of year.
The value of the coefficient for any week, $w$-call it $b_{w}$-equals ( -28.4 $\left.+1.40 w-0.0274 w^{2}\right)$. Table 9 shows the intraseasonal pattern of $b_{w}$. The average reduction in price associated with an increase of 100,000 pounds per week in California quantity ranges from 1 cent per pound in midseason to 3 cents at the beginning and end. The slopes here range from 30 to 90 times (average, 51 times) the slope of the annual function-a plausible result. Moreover, the coefficients never differ from the corresponding coefficients obtained with independent functions by more than two standard errors of the latter. This result is consistent with the idea that the two sets of coefficients would converge as sample size increased, but that for a given sample the generalized function contains less sampling error.
Also shown are coefficients of price flexibility at the artificial centroid for each week, as well as their reciprocals. The values cluster around those for annual demand, and again demand appears price-elastic at average volumes.

While $b_{w}$ varies over the season, for any given week the relation between Calavo price and California volume becomes linear. No shifting curvilinear relation, such as a power, or constant-elasticity, form was tried because, for one reason, Foote's test for nonlinearity, applied to residuals for weeks 12, 29, and 46, was once inconclusive and twice indicated linearity. Another reason was that curvilinearity would enormously complicate the treatment of intraseasonal maximization if elasticity varied over the season; also, an exponential form would produce absurd results if demand for any week turned out to be inelastic at every price.
If $n, \log y$, and independent California volume are assigned the values necessary to make the equation predict Calavo's actual prices in 1958-1959 without error, we obtain the intercepts labeled $a_{w, 1959}$ in table 9 , and the set of weekly Calavo demand functions shown in figure 5. Solid lines in the figure represent the ranges of observed total California quantities, and may indicate the range over which the linear relations are reasonable approximations. The dot on each line represents Calavo's 1958-1959 result.
Mean values of the variables over the period covered (1939-1940 through 1958-1959) were included in table 9 . Comparison of $\bar{c}_{w}$ with $b_{w}$ shows a consistent inverse relation; the greater the mean California quantity, the flatter was the demand curve. This suggests that what has so far been interpreted as intraseasonal shifts in demand may really represent movements along a single curvilinear relation. It is true that the $t$ ratios for the product terms in the generalized function far exceed usual critical values; but it is also the case that the standard errors of the regression coefficients ought to be adjusted for serial correlation in the residuals-which a Durbin-Watson test


Fig. 5. Weekly net regressions of Calavo price on Calavo quantity, with intercepts inferred from actual 1958-1959 prices.
indicates does exist-and in any event $t$ tests are only probabilistic. Consequently, another test was made.

A function was fitted that would be fixed through the season and curvilinear in California volume. The result, again based on data for the midmonth weeks of 1939-1940 through 1958-1959, was:

$$
\begin{array}{r}
\hat{p}=-47.9-12.85 \log c+1.14 n+29.4 \log y, \\
(t=9.59)(t=1.01)(t=12.11)
\end{array}\left\{\begin{array}{l}
R^{2}=0.443 \\
S_{1.234}=5.9
\end{array}\right\}
$$

In all respects this fit seems less satisfactory: much lower determination; a coefficient of log-income with a larger standard error and a value that differs markedly from the annual coefficient; a coefficient of non-California volume with a "wrong" sign and, furthermore, a low $t$ ratio. Presumably other curves would also appear less satisfactory (that is, other forms of function or functions using other data, such as weekly averages of price and quantity).

But why should demand shift during the season? Probably the answer lies in the changing availability of numerous competitive and complementary fresh fruits and vegetables. For reasons mentioned under "Usage," p. 714, the availability of such commodities was not incorporated in the demand function. "Week," however, may be serving as a proxy for regular seasonal variation in the absolute or relative prices and/or deliveries of those commodities. ("Week" may also be serving as a proxy for regular seasonal variation in the varietal and grade composition of Calavo's sales. No attempt was made to include the variety and grade indexes mentioned under "Qualities," p. 717, in the generalized function, because neither had even approached significance in either the annual or the separate-weeks analyses.) For certain commodities, deliveries, for example, to the Los Angeles metropolitan area (U. S. Department of Agriculture, Los Angeles Unloads of Fresh Fruits and Vegetables) typically peak about September-October, the time when avocado weekly demand is lowest, and/or trough about March-April, the time when avocado weekly demand is greatest. This is true of apples, figs, grapes, lettuce, pears, potatoes, and tomatoes, as well as beans, eggplant, peppers, persimmons, and pomegranates. Perhaps some or all of these commodities in the net are substitutes. For certain other commodities, Septem-ber-October is typically a trough and/or March-April a peak: artichokes, asparagus, bananas, cabbage, celery, grapefruit, oranges, pineapples, and radishes, as well as broccoli, cauliflower, onions, parsnips, green peas, rhubarb, and spinach. These may be complements. Or perhaps some obscure aggregate of these and other commodities moves systematically.

While it would be desirable to know what produces the intraseasonal shifting, the knowledge is not critical for present purposes provided the idea of intraseasonal shifts can be accepted on the basis of the statistical evidence plus mere conjectures about causation. Indeed, given this acceptance and provided that the pattern of intraseasonal variation in omitted variables is stable, our generalized function is in some respects more efficient precisely because variables have been omitted for which "week" serves as a proxy. Fewer degrees of freedom are lost; recalculating when additional data become available is easier; estimated prices become simpler to calculate; and predictions require guesses about the future values of fewer variables.

Variables Deleted. Two other variables were omitted only after trial. One was Calavo quantity during the previous week (see Appendix table 3) along with it times $w$ and $w^{2}$. The associated $t$ ratios were sufficient at the 5
per cent level. However, the $t$ ratios attaching to terms in current California quantity fell virtually to zero. Apparently the correlation of current and lagged quantities prevented distinguishing their effect on current price. The coefficient of correlation between lagged and current Calavo quantities was 0.971 ; that for lagged Calavo quantity and current California quantity (a series obtained by applying Calavo's weekly proportions to annual figures for Calavo plus independents) was 0.962 .

The other deletion was year. When it was included along with product terms of year times $w$ and $w^{2}$, the associated $t$ ratios were not encouraging. On the other hand, when year alone was included, its coefficient was significant at almost the 1 per cent level. Even so, including it increased $R^{2}$ by only 0.0075 -apparently because, without year, trend factors had been incorporated in the coefficient of log-income. This, in conjunction with unsatisfactory $t$ ratios in the annual and independent weekly analyses, led to its deletion.

## INTRASEASONAL MAXIMIZATION

We now have the minimum information needed to estimate what intraseasonal allocation of given total volumes would maximize net income to Calavo, and also what maximum fresh volume could profitably be sold.

Because demand varies over the season, optimal allocation involves price fluctuations. It is true that stable prices would be preferable on administrative grounds and for sales promotion and distributor support. If stability were practicable, it would be worth arguing whether these advantages would outweigh the greater revenues that are possible with instability. However, stability is quite infeasible in the face of uneven fruit maturity and competition from other handlers. The idea then is to cause the instability to occur in the way most favorable to Calavo (and presumably to continue to promote sales with price guarantees; no attempt was made in the present study to evaluate Calavo's use of this common device for sales promotion or to measure the difference in the cost of price guarantees that is implied by different intraseasonal allocations).

## Without Uncertainty or Maturity Problems

It is helpful to consider first the solution that would follow if the cost and demand equations were accepted as correct, if all uncontrollable variables were known for certain, and if maturities were no problem. The solution with uncertainties and storage limitations can then be discussed more intelligibly in terms of needed modifications.

The General Solution. Suppose the remaining part of the season extends from week $t$ through week 52. If everything were certain, we should know the production available to Calavo during the remainder of the season, the relation between Calavo unit handling costs and Calavo quantity by weeks, the relation between Calavo f.o.b. price and Calavo quantity by weeks, and the relation between Calavo net return (if any) per unit not sold fresh and the amount that Calavo diverts, by weeks.

The production still available $\left(A_{t}\right)$ would be total Calavo carry-over from the previous season and production for the present season $(A)$ minus the amounts already disposed of:

$$
\begin{equation*}
A_{t}=A-\sum_{w=1}^{t-1} q_{w}-\sum_{w=1}^{t-1} s_{w} \tag{1}
\end{equation*}
$$

where $\sum_{w=1}^{t-1} q_{w}$ and $\sum_{w=1}^{t-1} s_{w}$ represent the number of million pounds already disposed of in fresh and "surplus" markets, respectively.

The functions for unit handling costs, according to the section on "Handling Costs," p. 725, would be a set of identical curvilinear equations:

$$
\begin{equation*}
u_{w}=3.51 q_{w}^{-1}+3.0-0.5 q_{w}, w=t, t+1, \cdots, 52 \tag{2}
\end{equation*}
$$

The fresh-market demand functions, according to the section on "Weekly Demand," p. 735, would be a set of linear equations:

$$
\begin{equation*}
\hat{p}_{w}=a_{w}+b_{w} q_{w}, w=t, t+1, \cdots, 52 \tag{3}
\end{equation*}
$$

The parameters $a_{w}$ and $b_{w}$ would be known. With respect to $b_{w}$, suppose that the values are those obtained earlier:

$$
\begin{equation*}
b_{w}=-28.4+1.40 w-0.0274 w^{2}, w=t, t+1, \cdots, 52 \tag{4}
\end{equation*}
$$

With respect to $a_{w}$, suppose that the values are those that would have been needed, in conjunction with the $b_{w}$ 's, in order to have predicted Calavo's actual f.o.b. prices in 1958-1959 without error:

$$
\begin{equation*}
a_{w}=p_{w, 1959}-b_{w} q_{w, 1959}, w=t, t+1, \cdots, 52 . \tag{5}
\end{equation*}
$$

That is, the weekly demand functions would be the set of which 12 were shown in table 9 and figure 5 .

The relation between Calavo net return per unit diverted to nonfresh outlets and the amount diverted is, supposedly, independent of events in the fresh market, and is represented by the unspecified functions:

$$
\begin{equation*}
v_{w}=v_{w}\left(s_{w}\right), w=t, t+1, \cdots, 52 \tag{6}
\end{equation*}
$$

where $s_{w}$ designates million pounds of "surplus" diverted in week $w$ to processing, abandonment, or carry-over into next season, and $v_{w}$ represents cents per pound of net value from diversion.

Total revenue minus total handling costs for any week $w$ then would be:

$$
\begin{align*}
v_{w} s_{w}+q_{w}\left[a_{w}\right. & \left.+b_{w} q_{w}\right]-q_{w}\left[3.51 q_{w}^{-1}+3.0-0.5 q_{w}\right]  \tag{7}\\
& =v_{w} s_{w}-3.51+q_{w}\left[a_{w}-3.0\right]+q_{w}^{2}\left[b_{w}+0.5\right]
\end{align*}
$$

This amount represents income net of all costs except sales promotion.

Now net income realized in different weeks must be made comparable. Suppose that a week's revenues and expenses are converted to cash at the end of the week, and that a time preference, or interest rate, of 5.88 per cent compounded annually is appropriate. This figure is selected because it equals 5.72 per cent per year compounded weekly, which corresponds to a round number, 0.11 per cent, per week. Use $r_{w}$ to designate the amount by which a dollar of net income in week $w$ should be multiplied in order to raise it to its worth as of the end of the season. Then:

$$
\begin{equation*}
r_{w}=(1.0011)^{52-w} \tag{8}
\end{equation*}
$$

Now net income realized over a number of weeks can be expressed in terms of an equivalent end-of-season amount- $I$. This amount, for the part of the season beginning with week $t$, is:

$$
\begin{equation*}
I_{t}=\sum_{w=t}^{52}\left(r_{w} v_{w} s_{w}-3.51 r_{w}+r_{w} q_{w}\left[a_{w}-3.0\right]+r_{w} q_{w}^{2}\left[b_{w}+0.5\right]\right) \tag{9}
\end{equation*}
$$

where $I_{t}$ represents compounded seasonal income in units of $\$ 10,000$, since cents per pound are multiplied by number of million pounds.

Calavo's objective is taken to be the maximization of $I_{t}$, with the condition that not more (and not less) than the production still available may be sold or diverted (including carried-over) during the remainder of the season:

$$
\begin{equation*}
Q_{t}+S_{t}=\cdot A_{t} \tag{10}
\end{equation*}
$$

where $Q_{t}$ is a shorthand for $\sum_{w=t}^{52} q_{t}$ and $S_{t}$ is a shorthand for $\sum_{w=t}^{52} s_{w}$. Also, the amounts produced, sold fresh, and diverted, along with weekly prices and marginal handling costs, may not be less than zero.

The objective, plus the condition expressed by equation (10), is equivalent to maximizing the amount:

$$
\begin{equation*}
J_{t}=I_{t}+\lambda_{t}\left[A_{t}-Q_{t}-S_{t}\right] \tag{11}
\end{equation*}
$$

where $\lambda_{t}$ represents the number of dollars of potential net income in other weeks that would be sacrificed by selling an additional 100 pounds in any one week. This equation permits showing that the net impact on seasonal income of an increase in fresh quantity in any one week is the change in that week's income minus the sacrifice of income in other weeks:

$$
\begin{equation*}
\partial J_{t} / \partial q_{t}=r_{t}\left[a_{t}-3.0\right]+2 q_{t} r_{t}\left[b_{t}+0.5\right]-\lambda_{t} \tag{12,t}
\end{equation*}
$$

$$
\begin{equation*}
\partial J_{t} / \partial q_{52}=r_{52}\left[a_{52}-3.0\right]+2 q_{52} r_{52}\left[b_{52}+0.5\right]-\lambda_{t} \tag{12,52}
\end{equation*}
$$

$$
\begin{equation*}
\partial J_{t} / \partial q_{51}=r_{51}\left[a_{51}-3.0\right]+2 q_{51} r_{51}\left[b_{51}+0.5\right]-\lambda_{t} \tag{12,51}
\end{equation*}
$$

$$
\begin{equation*}
\partial J_{t} / \partial Q_{t}=-\lambda_{t} \tag{12,53}
\end{equation*}
$$

For $J_{t}$ to be maximized, it is necessary (although not sufficient) to adjust $q_{t}, q_{t}+_{1}, \ldots, q_{52}$ to the acceptable levels, $\bar{q}_{w}$, at which each of the above net impacts equals zero:

$$
\begin{equation*}
0 \leqq \bar{q}_{t}=\left(\lambda_{t}-r_{t}\left[a_{t}-3.0\right]\right) / 2 r_{t}\left[b_{t}+0.5\right] \leqq-a_{t} / b_{t} \tag{13,t}
\end{equation*}
$$

.

$$
\begin{align*}
& 0 \leqq \bar{q}_{51}=\left(\lambda_{t}-r_{51}\left[a_{51}-3.0\right]\right) / 2 r_{51}\left[b_{51}+0.5\right] \leqq-a_{51} / b_{51}  \tag{13,51}\\
& 0 \leqq \bar{q}_{52}=\left(\lambda_{t}-r_{52}\left[a_{52}-3.0\right]\right) / 2 r_{52}\left[b_{52}+0.5\right] \leqq-a_{52} / b_{52}
\end{align*}
$$

Solving these equations for $\bar{q}_{t}, \ldots, \bar{q}_{52}$ will give optimal weekly quantities in 1958-1959 for a given total fresh volume.

All values on the right side of equalities $(13 t)$ - $(13,52)$ are known except $\lambda_{t}$. To obtain an expression for $\lambda_{t}$, sum on each side of these equalities:

$$
\begin{equation*}
\sum_{w=t}^{52} \bar{q}_{w}=\lambda_{t} \sum_{w=t}^{52}\left(2 r_{w}\left[b_{w}+0.5\right]\right)^{-1}-\sum_{w=t}^{52}\left(\left[a_{w}-3.0\right] / 2\left[b_{w}+0.5\right]\right) \tag{14}
\end{equation*}
$$

Now substitute $Q_{t}$ for $\sum_{w=t}^{52} q_{w}$ and solve for $\lambda_{t}$ :
(15a) $\quad \lambda_{t}=\left\{Q_{t}+\sum_{w=t}^{52}\left(\left[a_{w}-3.0\right] / 2\left[b_{w}+0.5\right]\right)\right\} / \sum_{w=t}^{52}\left(2 r_{w}\left[b_{w}+0.5\right]\right)^{-1}$

$$
\min _{w \geqq t}\left(r_{w}\left[a_{w}-3.0\right]-2 a_{w} r_{w}\left[b_{w}+0.5\right] / b_{w}\right) \leqq \lambda_{t} \leqq \min _{w \geqq t}\left(r_{w}\left[a_{w}-3.0\right]\right)
$$

By doing the arithmetic for the 1958-1959 season as a whole-that is, for $t=1$, the following is obtained:

$$
\begin{gather*}
\lambda_{1}=\left(Q_{1}-45.2185\right) /-1.7553=25.761-0.5697 Q_{1}  \tag{15b}\\
\left\{\begin{array}{c}
r_{13}\left[a_{13}-3.0\right]-2 a_{13} r_{13}\left[b_{13}+0.5\right] / b_{13} \\
=-20.3 \leqq \lambda_{1} \leqq r_{13}\left[a_{13}-3.0\right]=18.1 \\
80.9 \geqq Q_{1} \geqq 13.4
\end{array}\right\}
\end{gather*}
$$

This equation says that $\lambda_{1}$ is a linear function of $Q_{1}$ for values of $Q_{1}$ within a certain range. The value of $Q_{1}$ may not be less than 13.4. Below this volume, $\lambda_{1}$ would exceed $\$ 18.1$ per hundred pounds, which is the value of the smallest intercept among the equations for weekly marginal net income, including interest. The restriction is necessary because equality (15a) was derived without explicit ${ }^{\text {recegnition }}$ of the conditions attached to equalities (13)-that negative quantities may not be sold in any week; for fresh sales slightly below 13.4 million pounds, equality $(13,13)$ would imply choosing a negative quan-
tity for week 13. Since all negative quantities are excluded, the value of $\lambda_{1}$ associated with values of $Q_{1}$ that are progressively less than 13.4 will be progressively greater than equality (15b) implies. Working up from $Q_{1}=0$ to $Q_{1}=13.4, \lambda_{1}$ is given by a series of linear segments the slopes of which are successively less negative.

On the other hand, for a value of $Q_{1}$ slightly greater than $80.9, \lambda_{1}$ would be less than minus 20.3 . This would imply a value for $\bar{q}_{13}$ in excess of the upper bound on equality $(13,13)$; so large a quantity would, according to equation $(3,13)$, produce a negative value of $\hat{p}_{13}$. No information is available about this range.


Fig. 6. Actual versus optimal Calavo quantities, by weeks, 1958-1959.
With equations (13) and (15b), for any acceptable value of $Q_{1}$, the weekly quantities that would have maximized seasonal net income from fresh sales in 1958-1959 can be calculated. If net returns from processing and carry-over were negligible, the same quantities would have maximized total seasonal income for that available volume. For values of $Q_{1}$ outside the acceptable range, more complicated programming techniques would need to be employed in order to calculate optimal weekly quantities.

Solution for 1959 Volume. One value of $Q$ (the subscript 1 is hereafter omitted) of particular interest is the volume actually sold in 1958-1959: 46.172 million pounds. Equation (15b) shows that the associated value of $\lambda$ is minus 54 cents per 100 pounds; that is, attainable seasonal net income would have
been sold, even if "the entire 46.172 million pounds had been allocated optimally. To find the most profitable way of allocating the 46.172 million pounds, equations (13) are solved for $\bar{q}_{w}$. The optimal quantities are shown in figure 6 by the path labeled "optimal for $\hat{Q}_{1}=Q=46.172$." The estimated prices that would have moved these quantities, calculated from equations (3), are shown in figure 7. For comparison, Calavo's actual quantities and prices also are plotted.


Fig. 7. Actual versus optimal Calavo prices, by weeks, 1958-1959.
How much seasonal net income was lost because the 46.172 million pounds were not optimally allocated? If seasonal net income is calculated from equation (9) for the actual pattern, the answer is $\$ 3,745,000$. (Further calculations disclose that this figure represents f.o.b. sales revenues of $\$ 6,609,000,{ }^{15}$ minus handling costs of $\$ 2,962,000,{ }^{16}$ plus imputed interest of $\$ 98,000$.) If the optimal

[^10]quantities are inserted into equation (9), the answer is $\$ 4,046,000$. The difference is $\$ 301,000$, or 8.0 per cent, or 0.65 cent per pound.

Solution for Maximum Fresh Income. It would be interesting to know how much of the 46.172 million pounds could profitably have been sold fresh if processing or carry-over were a competing alternative, but we do not have the necessary information-equations (6). However, an upper limit for this amount can be found. The limit is the largest volume that could profitably have been sold fresh if availability were so large or diversion so profitless that fresh-market income could be maximized separately.

In this event, $\lambda=0$, a condition obtained when equation $(12,53)$ is set equal to zero. Hence the upper limit is found by setting the value of $\lambda$ given in equation ( 15 b ) equal to zero and solving for $Q .{ }^{17}$ The answer is 45.2185 million pounds. When availability exceeded this figure, seasonal income could have been maximized only if the excess was diverted to non-fresh markets. The associated weekly quantities, calculated from equations (13), if shown in figure 6 would lie slightly below those shown for $\widehat{Q}_{1}=Q=46.172$. The associated prices, calculated from equations (3), would appear in figure 7 slightly above those shown for $\hat{Q}_{1}=Q=46.172$.

How much could seasonal income have been increased by optimal allocation of 45.2185 million pounds? If seasonal net income is calculated from equation (9) for $Q=45.2185$, the answer is $\$ 4,049,000$. This is $\$ 3,000$, or 0.08 per cent, greater than the figure for optimal allocation of 46.172 , and $\$ 304,000$, or 8.1 per cent, greater than the figure for the quantities that Calavo actually sold.

General Relation of Income and Price to Volume. Actually, it was unnecessary to calculate the income attainable with 45.2185 million pounds from equation (9). A formula can be derived which allows direct calculation of the seasonal net income attainable from fresh sales, $\bar{I}$, for any relevant value of $Q . \lambda$ is the rate of change of $\bar{I}$ with respect to $Q$, and the value of $\lambda$ is obtained from equation (15b). The function of $Q$ that would have equation (15b) as its first derivative is:

$$
\begin{equation*}
\bar{I}=K_{1}+25.761 Q-0.28485 Q^{2}, \quad 13.4 \leqq Q \leqq 80.9 \tag{16}
\end{equation*}
$$

where $\bar{I}$ represents attainable income in units of $\$ 10,000$ and $K_{1}$ is an unknown constant. Since $\bar{I}=\$ 4,046,000$ for $Q=46.172$, we can solve for $K_{1}$ :

$$
\begin{equation*}
K_{1}=404.6-25.761[46.172]+0.28485[2,131.85]=-177.6 \tag{17}
\end{equation*}
$$

Equation (16) divided by $A$ or by $Q$ gives the formula for attainable income from fresh sales per pound produced (plus carried in) or per pound sold fresh (in cents per pound):

[^11]\[

$$
\begin{gather*}
\bar{I} / A=-177.6 / A+25.761 Q / A-0.28485 Q^{2} / A  \tag{18}\\
13.4 \leqq Q \leqq 80.9 \\
\bar{I} / Q=-177.6 / Q+25.761-0.28485 Q  \tag{19}\\
13.4 \leqq Q \leqq 80.9
\end{gather*}
$$
\]

A general expression for the maximum attainable fresh sales revenue per pound sold fresh would be:

$$
\begin{align*}
\bar{P}= & Q^{-1} \int d Q\left\{Q+\sum_{w=1}^{52}\left(a_{w} / 2 b_{w}\right)\right\} / \sum_{w=1}^{52}\left(2 b_{w}\right)^{-1}  \tag{20}\\
= & Q^{-1} \int d Q\{Q-48.8109\} /-1.7397 \\
= & K_{2} / Q+28.06-0.2874 Q \\
= & 11.2 / Q+28.06-0.2874 Q \\
& 13.4 \leqq Q \leqq 84.2
\end{align*}
$$

where $\bar{P}$ represents attainable season-average price in cents per pound, with no interest added. The derivation is exactly parallel to that underlying equation (18), and need not be repeated. $\bar{P}$ is shown in figure 4 by the curve labeled "maximum attainable."

Advisability of Diverting Surplus. It should be mentioned that the highest attainable level of seasonal income from fresh sales was not necessarily the appropriate level for Calavo to seek, quite aside from any net returns available from processing or carry-over, from the costs of picking and hauling, and from the costs of programming. When members' production exceeds the volume for which seasonal net income from fresh sales would be greatest, maximizing net income requires that Calavo divert some production out of the fresh market, and this has disadvantages as well as advantages.

Attainable revenue per pound sold fresh would increase, of course. It would increase, according to equation (20), by:

$$
\begin{aligned}
11.2 /(A-S)+ & 28.06-0.2874[A-S]-\{11.2 / A \\
& +28.06-0.2874 A\} \\
= & 11.2\left[(A-S)^{-1}-A^{-1}\right]+0.2874 S
\end{aligned}
$$

or by 0.28 cent per pound where $A=46.172$ and $A-S=45.2185$.
However, attainable net income per pound sold fresh would not increase that much. According to equation (19), it would increase by:

$$
\begin{aligned}
-177.6 /(A-S)+25.761-0.28485[A-S] & -\{-177.6 / A \\
& +25.761-0.28485 A\} \\
= & 0.28485 S-177.6\left[(A-S)^{-1}-A^{-1}\right\rfloor
\end{aligned}
$$

or by 0.19 cent per pound where $A=46.172$ and $A-S=45.2185$.

Attainable net income from fresh sales per pound produced (plus carried in) would increase even less. According to equation (18), it would increase by:

$$
\begin{aligned}
-177.6 / A+ & 25.761[A-S] / A-0.28485[A-S]^{2} / A \\
& \quad-\{-177.6 / A+25.761-0.28485 A\} \\
= & 0.5697[1-45.2185 / A] S-0.28485 S^{2} / A
\end{aligned}
$$

or by 0.007 cent per pound where $A=46.172$ and $A-S=45.2185$. If processing yields only negligible net income, it is this 0.007 cent per pound that must be compared with the disadvantage involved.

The disadvantage is that nonmembers and/or competing handlers also would benefit. Unilateral diversion by Calavo would lead to higher wholesale prices not just for Calavo production, but for other California production as well. Indeed, independents' gain per pound would be the full amount of the price increase, since they would reap the benefit on their entire production. A proprietary concern might disregard this effect, but a marketing coöperative will not. The tendency among coöperatives is to maximize members' net returns only so long as this is consistent with appearing competitive with independent handlers; workers in the field of agricultural coöperation may not have recognized the extent to which viability for the organization and tenure for the management dictate this constraint. Since Calavo has been hard pressed by competition in recent years (see "Concentration and Collaboration Among Handlers," p. 713), it would be understandably reluctant to divert "surpluses" unless it stood to gain more than independents would.

The previous calculations do not provide a direct estimate of independents' gain. However, the coefficient of California quantity in Calavo's annual demand equation is indicative. The coefficient says that Calavo's season average price has increased an average of 0.327 cent per pound for each million pounds reduction in annual California fresh sales. This corresponds to an increase of 0.37 cent per pound if Calavo had diverted 0.953 of its 46.172 million pounds. The increase in independents' prices would need to be less than one fiftieth as much if their gain were not to exceed Calavo's.

Would some smaller diversion be more likely to yield greater income to Calavo than to independents? If $A-S$ is substituted for $Q$ in equation (18), and if we differentiate with respect to $S$, the rate of change of Calavo's attainable income per pound available with respect to surplus is:

$$
\begin{equation*}
\partial(\bar{I} / A) / \partial S=0.5697-25.761 / A-0.5697 S / A \tag{21a}
\end{equation*}
$$

With $A=46.172$, this implies:

$$
\begin{equation*}
\partial(\bar{I} / A) / \partial S=0.012-0.0123 S \tag{21b}
\end{equation*}
$$

That is, Calavo's attainable income per pound available would increase at a rate no greater than 0.012 cent per pound per million pounds diverted, no matter how little was diverted. Hence, the rate of change of independents'
prices would need to be less than 0.012 . But this means that the rate of change of independents' prices would need to be less than one twenty-fifth of 0.327 . This is still very unlikely, and we may conclude that no amount of unilateral diversion would have yielded greater attainable income to Calavo than to independents. Furthermore, the relative benefits to members would tend to diminish even more over time if the higher prices attracted increased competitive production.

## With Uncertainty

The uncertainties relevant to the solution expressed by equations (13) and (15) should be divided into two groups-those pertaining to weekly demands and handling costs and those pertaining to forecasted total fresh volume. There is every reason to suppose that mistakes about one are independent of mistakes about the other. As a result, each can be approached as if the other were not a problem.

Uncertain Marginal Incomes. Calculation of the optimal quantities indicated in equations (13) presupposes knowledge of the parameters of weekly demand and cost functions.

The demand parameters used above stem from the generalized weekly demand function. For reasons mentioned under "Demand for Calavo Avocados," p. 729 , this function is imperfectly reliable. Part of the unreliability could be avoided for a season already past via equations (5). These begged the question of the true effect of non-Calavo quantities and of nonagricultural income. Even hindsight, however, does not permit asserting with certainty the values of $b_{w}$, and therefore the prices that would have occurred if different Calavo quantities had been sold; and the error of estimate presumably would be even larger if we undertook programming for a coming season.

Similarly, even with adequate data an accurate handling cost function could not be obtained. The upshot is that the impact on seasonal income of variations in a week's quantity can only be an estimate, subject to an unknowable error.

Recognizing this uncertainty, however, is no reason to modify the stated solution. There is no apparent reason for choosing quantities different from those indicated by the latest estimates corresponding to equations (13). The uncertainty should, however, prepare us for unexpected outcomes. The symbol $I$ will continue to represent compounded seasonal income, but it will now mean likely income, or the income that the equations indicate.

Uncertain Total Volume. Calculation of optimal quantities from equations (13) also presupposes a value for $\lambda_{t}$. The calculation of $\lambda_{t}$ indicated in equation (15) applies to a situation where $Q_{t}$ is known. In a planning context, this is not the case. Available production often differs substantially from that predicted, and there is no reason why diversions should simply absorb the difference.

The first reaction is simply to substitute for $Q_{t}$ in equation (15) the best estimate of it available at the time of planning, $\hat{Q}_{t}$, and proceed as if the uncertainty attaching to $\hat{Q}_{t}$ implied only that periodic recalculation of optimal
weekly quantities is now appropriate as the season progresses. The idea would be to calculate (or abide by) the initial weekly program for only, for example, 13 weeks. At the end of the first quarter, by means of an improved estimate of $Q_{14}, \bar{q}_{14}$ through, for example, $\bar{q}_{26}$ would be calculated. Later, by an improved estimate of $Q_{27}, \bar{q}_{27}$ through, for example, $\bar{q}_{39}$ would be calculated. Finally, $\bar{q}_{40}$ through $\bar{q}_{52}$ would be calculated, but a difference between $\hat{Q}_{40}$ and $Q_{40}$ might arise because of informal departures from the calculated program during the final quarter.

Such periodic recalculation is appropriate, but a simple substitution of $\hat{Q}_{t}$ for $Q_{t}$ is not.

Predicted vs. Planned Volume. The substitution of $\hat{Q}_{t}$ for $Q_{t}$ would beg two important questions.

First, what, in the context of uncertainty, is Calavo's objective? Let us tentatively suppose it to be the maximization of the expected value of seasonal income, subject to a maximum rate of increase in risk. By "the expected value of seasonal income," or $E(I)$, is meant the sum of each possible value of income times its likelihood of occurrence. ${ }^{18}$

Second, what figure, when substituted for $Q_{t}$ in equation (15), would produce answers for $\bar{q}_{t}$ through, for example, $\bar{q}_{t+12}$ that would maximize $E(I)$ ? Let us call the figure $\bar{\rho}_{t} \hat{Q}_{t}$. It is the optimal total from which to make weekly allocations. That is, $\rho_{t}$ represents a ratio of planned to expected, or predicted, total fresh sales, and $\bar{\rho}_{\iota}$ represents the particular ratio that would maximize $E(I)$. The procedure outlined above implies that $\bar{\rho}_{t}$ invariably equals unity, but this is by no means obvious.
Suppose that $\hat{Q}_{1}$ is less than 45.2185 , so that, if $\rho_{1}=1$, all initially planned quantities would be less than the maximum quantities that could profitably be sold. (The argument is reversed if $\hat{Q}_{1}>45.2185$.) If the prediction proves to be wrong by some amount, it matters whether it proves to be too high or too low. It is true that in either event the actual volume will be nonoptimally allocated from an ex post viewpoint. But if the prediction proves high, actual volume will be smaller than the predicted amount, whereas if the prediction proves low, actual volume will be larger than the predicted amount. With $\hat{Q}_{1}<45.2185$, the smaller volume would reduce income below that which would result if the prediction proved accurate, whereas the larger volume would raise income above this level-provided the larger volume were not too great.

This asymmetry makes it possible (where very large volumes are either unlikely or not very costly) to reduce risk by lessening the reduction in income that would follow from a high prediction. This can be done by making sure that, if a volume below that predicted does materialize, its allocation would be closer to optimal. This means reducing the quantities sold in early weeks below those that would be optimum if the prediction proved accurate. Such

[^12]a reduction is accomplished by planning quantities in the early weeks as if a smaller total volume were expected-that is, by choosing a value for $\rho_{1}$ below unity.

Of course, there is an offset. If the prediction proves low, allocation will be further from optimal than if $\rho_{1}=1$. With $\hat{Q}_{1}<45.2185$, however, there is room for $\rho_{1}$ to go some amount below unity before every low prediction would be made also to imply lower income.

In terms of the impact on $E(I)$, however, increasing the incomes that high predictions will produce may or may not outweigh the corresponding decrease in low-prediction incomes. The net result depends on how income responds to variations in allocation. Risk could perhaps be reduced only if $E(I)$ were reduced simultaneously. If so, the desirability depends on the amounts involved and on the relative strength of risk aversion and desire for income. Let us concentrate initially on maximizing $E(I)$.

The significance of the level of $\rho_{1}$ is that it influences the value of $I$ that will result from the value that $Q$ turns out to have. For a given value of $\rho_{1} \hat{Q}_{1}$, a value of $I$ corresponds to each value of $Q . E(I)$ for the given value of $\rho_{1} \hat{Q}_{1}$ then is the sum of each of these values of $I$ times the likelihood of the associated value or values of $Q$. To maximize $E(I)$, we must select the value of $\rho_{1} \hat{Q}_{1}$ for which $E(I)$ is greatest.

Several problems are involved: how does $I$ respond to $Q$ at particular values of $\rho_{1} \hat{Q}_{1}$; what likelihood attaches to various values of $Q$, so that $E(I)$ can be estimated for the various values of $\rho_{1} \widehat{Q}_{1}$; and what is the value of $\rho_{1}$ for which $E(I)$ is greatest.

The Income Response Surface. The character of the association of $I$ with $\rho_{1} \hat{Q}_{1}$ and $Q$ in 1958-1959 is shown in figure 8.

General Characteristics. To understand figure 8, suppose, first, that $\rho_{1} \hat{Q}_{1}$ is some amount less than 45.2185 . Then, as $Q$ rises from zero toward $\rho_{1} \hat{Q}_{1}, I$ increases because the quantities sold during the later weeks increase and, with $\rho_{1} \widehat{Q}_{1}<45.2185$, the later weeks could profitably absorb at least as much as the quantities initially contemplated. For $Q=\rho_{1} \widehat{Q}_{1}$, the value of $I$ is given by equation (16), since there is no difference during the season among actual, initially planned, and optimum quantities. As $Q$ rises further, $I$ continues to increase until it peaks at a value of $Q$ greater than $\rho_{1} \hat{Q}_{1}$ and less than 45.2185; $I$ declines as $Q$ rises further. The peak occurs at $Q>\rho_{1} \hat{Q}_{1}$ (that is, beyond the 45 -degree line in the $\rho_{1} \hat{Q}_{1}-Q$ plane) because, with $\rho_{1} \hat{Q}_{1}<45.2185$, quantities larger than those initially contemplated could profitably be sold during the later weeks. The peak occurs at $Q<45.2185$ because as much as 45.2185 could profitably be sold only if planned for from the start of the season. However, the larger was $\rho_{1} \hat{Q}_{1}$, the closer to 45.2185 will be the value of $Q$ at which $I$ peaks, since larger quantities will have been planned and actually sold during the early weeks.

Second, suppose $\rho_{1} \hat{Q}_{1}>45.2185$. As $Q$ rises from zero, $I$ increases, reaching a peak at a value of $Q$ greater than 45.2185 and less than $\rho_{1} \hat{Q}_{1}$; as $Q$ rises further, $I$ declines, its value for $Q=\rho_{1} \hat{Q}_{1}$ being given by equation (16). The


Fig. 8. 1958-1959 relation of income to actual and planned volume.
peak occurs at $Q<\rho_{1} \hat{Q}_{1}$ (that is, before the 45 -degree line) because, with $\rho_{1} Q_{1}>45.2185$, quantities larger than can profitably be absorbed will have been contemplated initially for every week, and income would be greater if less than this was actually sold during the later weeks. The peak will occur at $Q>45.2185$ because quantities larger than those optimal for $Q=45.2185$ will have been planned and actually sold during the early weeks, so that the
season's total will exceed 45.2185 even if only the quantities optimal for $Q=$ 45.2185 are sold in the later weeks. Furthermore, the larger was $\rho_{1} \hat{Q}_{1}$, the more above 45.2185 will be the value of $Q$ at which $I$ peaks, since larger quantities will have been sold during the early weeks and therefore the larger must be the season's total in order to sell the quantities optimal with $Q=45.2185$ in the later weeks.
Third, suppose $\rho_{1} \hat{Q}_{1}=45.2185$. Then $I$ increases as $Q$ rises toward $\rho_{1} \hat{Q}_{1}$. $I$ peaks at this value of $Q$ (that is, at the 45 -degree line) and declines as $Q$ rises further. The value of $I$ for $Q=\rho_{1} \hat{Q}_{1}$ is, once again, given by equation (16); the value of $I$ for any point on the 45 -degree line is given by equation (16). The 45 -degree line has significance also in that (reading this time from north to south instead of from west to east) it indicates the value of $\rho_{1} \hat{Q}_{1}$ at which $I$ peaks for a given value of $Q$ : income is greatest when any given value of $Q$ is planned from the start.

Only the general nature of the dependence of $I$ on $Q$ for any given value of $\rho_{1} \widehat{Q}_{1}$ has been considered. To obtain information about the rate at which $I$ declines as $Q$ rises or falls from the value at which $I$ is greatest for that value of $\rho_{1} \hat{Q}_{1}$ various values of $I$ must be calculated.

Calculated Values. The calculations require four assumptions. The first pertains to how often optimal quantities are recalculated on the basis of revised estimates of total volume. Let us assume that programs are recalculated quarterly.
The second pertains to the rate at which the revised estimates of total volume approach the actual amount. In accordance with the discussion on predictability of production (p.000), let us assume that the error in a forecast of $Q$ made at the beginning of the second or third quarters is less than the error in the initial forecast in proportion to the percentage of $Q$ that turns out to have been allocated (sold) in previous quarters.

The third pertains to how $Q_{40}-\hat{Q}_{40}$ will be allocated over the final quarter. Let us assume that no such discrepancy arises; that is, fourth-quarter sales will be an optimal allocation of whatever volume proves to be left-that is, of $Q-\sum_{w=1}^{39} q_{w}$.

The fourth pertains to what values $\bar{\rho}_{14}$ and $\bar{\rho}_{27}$ will have. The problem here is that the optimal values of $\rho_{14}$ and $\rho_{27}$ depend in turn on what value of $\rho_{1}$ is chosen. The interdependence implies that we should solve for all of them (as well as for an optima number of ${ }^{\prime}$ 's-that is, an optimal number of recalculations) simultaneously. To do so, however, would expand the calculations exponentially. For present purposes, let us assume that $\bar{\rho}_{14}$ and $\bar{\rho}_{27}$ equal $\bar{\rho}_{1}$. Then any value of $\rho_{1}$ for which $E(I)$ is calculated becomes also the value assigned to $\rho_{14}$ and $\rho_{27}$, and the subscripts may hereafter be omitted.

Assumption four is not so arbitrary as it appears. The purpose of having $\rho_{1}<1$ is to lessen the loss of income that would follow from selling belowpredicted quantities later in the season. If this is desirable at the start of the season, presumably it will continue to be desirable in succeeding quarters.
Table 10
1958-1959 RELATION OF INCOME TO VOLUME FOR SELECTED PLANNING RATIOS

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Planning ratio: <br> $\rho$ | Postulated volume: Q | Net income: <br> I | Estimated seasonal volume as of: |  |  |  | Marginal profits during: |  |  |  | Likelihood of outcome $L(Q)$ | Discounted outcome:$I \cdot L(Q)$ | $\begin{aligned} & \text { Devia- } \\ & \text { tions: } \\ & I-E(I) \end{aligned}$ | $\begin{gathered} \text { Dis- } \\ \text { counted } \\ \text { absolute } \\ \text { devia- } \\ \text { tions: } \\ L(Q): \\ \|I-E(I)\| \end{gathered}$ |
|  |  |  | Week 1: <br> $\hat{Q}_{1}$ | $\begin{gathered} \text { Week 14: } \\ \hat{Q}_{14}+\sum_{w=1}^{13} q_{w} \end{gathered}$ | $\begin{aligned} & \text { Week 27: } \\ & \hat{Q}_{27}+\sum_{w=1}^{26} q_{w} \end{aligned}$ | Week 40: $\hat{Q}_{40}+\sum_{w=1}^{39} q_{w}$ | Quarter 1: <br> $\lambda_{1}$ | Quarter 2: <br> $\lambda_{14}$ | Quarter 3: <br> $\lambda_{27}$ | Quarter 4: <br> $\lambda_{40}$ |  |  |  |  |
|  | million $l b$. | 810,000 | million lb. | million lb. | million lb. | million $l$ b. | \$/100 lb. | 8/100 lb. | \$/100 lb. | \$/100 lb. |  | \$10,000 | \$10,000 | \$10,000 |
| 0.99 | 33.828 | 354.5 | 40.000 | 38.609 | 36.531 | 33.828 | 3.20 | 4.11 | 6.37 | 14.53 | 0.1456 | 51.6 | -28.6 | 4.2 |
| 0.99 | 36.914 | 382.0 | 40.000 | 39.363 | 38.386 | 36.914 | 3.20 | 3.59 | 4.58 | 8.71 | 0.3544 | 135.4 | - 1.2 | 0.4 |
| 0.99 | 43.086 | 397.2 | 40.000 | 40.546 | 41.416 | 43.086 | 3.20 | 2.77 | 1.63 | -4.46 | 0.3544 | 140.8 | 14.0 | 5.0 |
| 0.99 . | 46.172 | 380.4 | 40.000 | 41.018 | 42.688 | 46.172 | 3.20 | 2.44 | 0.40 | -11.64 | 0.1456 | 55.4 | $-2.7$ | 0.4 |
|  |  |  |  |  |  |  |  |  |  |  | 1.0000 | 383.2 |  | 10.0 |
| 1.00 | 33.828 | 352.8 | 40.000 | 38.596 | 36.502 | 33.828 | 2.97 | 3.95 | 6.39 | 15.02 | 0.1456 | 51.4 | -30.7 | 4.5 |
| 1.00 | 36.914 | 381.1 | 40.000 | 39.357 | 38.368 | 36.914 | 2.97 | 3.42 | 4.57 | 9.27 | 0.3544 | 135.1 | - 2.5 | 0.9 |
| 1.00 | 43.086 | 398.7 | 40.000 | 40.551 | 41.429 | 46.172 | 2.97 | 2.25 | 1.57 | - 3.78 | 0.3544 | 141.3 | 15.1 | 5.3 |
| 1.00 | 46.172 | 383.5 | 40.000 | 41.029 | 42.693 |  | 2.97 |  | 0.32 | -10.91 | 0.1456 | 55.8 | 0.0 | 0.0 |
|  |  |  |  |  |  |  |  |  |  |  | 1.0000 | 383.5 |  | 10.7 |
| 1.01. | 33.828 | 351.0 | 40.000 | 38.582 | 36.472 | 33.828 | 2.75 | 3.80 | 6.41 | 15.50 | 0.1456 | 51.1 | -32.7 | 4.8 |
| 1.01 | 36.914 | 380.0 | 40.000 | 39.350 | 38.358 | 36.914 | 2.75 | 3.26 | 4.56 | 9.83 | 0.3544 | 134.7 | - 3.8 | 1.3 |
| 1.01 | 43.086 | 399.9 | 40.000 | 40.556 | 41.442 | 43.086 | 2.75 | 2.41 | 1.51 | - 3.11 | 0.3544 | 141.7 | 16.2 | 5.7 |
| 1.01 . | 46.172 | 386.4 | 40.000 | 41.039 | 42.717 | 46.172 | 2.75 | 2.07 | 0.24 | -10.18 | 0.1456 | 56.3 | 2.6 | 0.4 |
|  |  |  |  |  |  |  |  |  |  |  | 1.0000 | 383.7 |  | 12.2 |
| 1.017 | 33.828 | 349.7 | 40.000 | $\ldots$ | $\ldots$ | 33.828 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.1456 | 50.9 | -34.1 | 5.0 |
| 1.017 | 36.914 | 379.1 | 40.000 | ...... | $\ldots$ | 36.914 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.3544 | 134.3 | -4.7 | 1.7 |
| 1.017 | 43.086 | 400.7 | 40.000 | $\ldots$ | ...... | 43.086 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.3544 | 142.0 | 16.9 | 6.0 |
| 1.017 | 46.172 | 388.3 | 40.000 | $\ldots$ | $\ldots$ | 46.172 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.1456 | 56.5 | 4.5 | 0.7 |
|  |  |  |  |  |  |  |  |  |  |  | 1.0000 | 383.8 |  | 13.3 |
| 1.00 | 40.000 | 397.1 | 40.000 | 40.000 | 40.000 | 40.000 | 2.97 | 2.97 | 2.97 | 2.97 | ...... | $\ldots$ | $\ldots$ | $\ldots$ |
| 1.00 | 45.219 | 404.9 | 45.219 | 45.219 | 45.219 | 45.219 | 0 | 0 | 0 | 0 | ...... | $\ldots$ | $\ldots$ | $\ldots$ |
| 1.00 | 46.172 | 404.6 | 46.172 | 46.172 | 46.172 | 46.172 | -0.54 | -0.54 | -0.54 | -0.54 |  |  | $\ldots$. |  |

Given these assumptions, a general expression can be obtained showing the dependence of $I$ on $\rho \hat{Q}_{1}$ and $Q$ :

$$
\begin{equation*}
I=\beta_{1}+\beta_{2} \rho \hat{Q}_{1}+\cdots+\beta_{n}\left(\rho \hat{Q}_{1}\right)^{6} / Q^{4} \tag{22}
\end{equation*}
$$

Calculation of the $\beta$ 's, however, would have involved a prodigious effort. Instead, the value of $I$ was calculated directly from equations (13), (15), and (9) for 12 combinations of $\rho \hat{Q}_{1}$ and $Q$. The value of $I$ that would result from each of four values of $Q$ was calculated for three values of $\rho: 0.99,1.00$, and 1.01. The calculations pertain to $\hat{Q}_{1}=40$; this was the rough preseason estimate of Calavo production in 1958-1959.

The four values of $Q$ were selected arbitrarily. One was the volume actually sold during 1958-1959: 46.172. Since this amount was substantially (15.4 per cent) higher than the original prediction, a second figure was chosen that was higher than the prediction by only half as much. Then two figures were chosen that would be the same percentages below the prediction.

The results are summarized in columns (1) to (3) of table 10 opposite $\rho$ values of $0.99,1.00$, and 1.01 , and are shown in figure 8 . Columns (4) to (7) are also of interest because they show the associated paths of estimated total volume. Columns (8) to (11) show the values of $\lambda_{t}$ according to which the weekly quantities in each quarter were determined.

Expected Income with Different Planning Ratios. The next problem concerns what likelihoods to attach to each of the four postulated values of $Q$. Still following the discussion on predictability of production (p. 723), let us suppose that the likelihoods correspond to ordinates of a normal probability distribution with a mean equal to the prediction and a standard deviation equal to 10 per cent of the prediction. Then the four values of $Q$ represent points differing from the mean by $-1.54,-0.77,+0.77$, and +1.54 standard deviations.

The associated ordinates, taken from standard tables, are 0.1219, 0.2966, 0.2966 , and 0.1219 . To obtain the likelihoods, multiply each of these numbers by $1.0 / 0.8370$, so that the sum equals unity. The result is $0.1456,0.3544$, 0.3544 , and 0.1456 (column (12), table 10).

The income associated with each value of $Q$, multiplied by the associated likelihood, equals the figures in column (13) of table 10. Summing these discounted incomes for any value of $\rho$ gives an estimate of $E(I)$ for that planning ratio. As indicated in table 10, the sum increases as $\rho$ rises from 0.99 to 1.00 to 1.01 . Compared to $E(I)$ with $\rho=1.00(\$ 3,835,000), E(I)$ with $\rho=0.99$ $(\$ 3,832,000)$ is 99.91 per cent $E(I)$ with $\rho=1.01(\$ 3,837,000)$ is 100.05 per cent.

The positive association of $E(I)$ with $\rho$ is central. It results from the fact that, as $\rho$ rises, the income that would result from a higher-than-expected value of $Q$ increases by more than the income that would result from an equally likely lower-than-expected value of $Q$ decreases. Thus, as $\rho$ rises from 1.00 to 1.01 , the income associated with $Q=46.172$ rises from $\$ 3,835,000$ to $\$ 3,864,000$, whereas the income associated with the equally likely $Q=33.828$ merely declines from $\$ 3,528,000$ to $\$ 3,510,000$.


Fig. 9. 1958-1959 relation of expected income, minimum likely income, and mean deviation to planning ratio.
$E(I)$ would not increase with $\rho$ indefinitely. Ultimately the decreases would outweigh the increases. The turning point occurs at the place called $\bar{\rho}$.

To estimate $\bar{\rho}$, we must learn how the income associated with each of the values of $Q$ would respond as $\rho$ rose above 1.01 . This response can be estimated by fitting a function to the three incomes associated with each value of $Q$, and then extrapolating.

Inspection of column (3) of table 10 suggested that straight-line functions would not be satisfactory. The first difference of a straight line is a constant. In contrast, the first differences for $Q=33.828$ are -1.73 and -1.80 ; for $Q=36.914,-0.97$ and -1.10 ; for $Q=43.086,1.43$ and 1.24 ; and for $Q=$ $46.172,3.07$ and 2.82 . Hence, parabolas were fitted. The results were as follows (with $I$ again expressed in units of $\$ 10,000$ ):

$$
\begin{align*}
& Q=33.828: I=184.4928+512.825 \rho-344.5 \rho^{2}  \tag{23}\\
& Q=36.914: I=-158.0880+1,182.15 \rho-643.0 \rho^{2}  \tag{24}\\
& Q=43.086: I=-696.5914+2,057.25 \rho-962.0 \rho^{2}  \tag{25}\\
& Q=46.172: I=-1,127.8226+2,728.37 \rho-1,217.0 \rho^{2} \tag{26}
\end{align*}
$$

By multiplying each of these four equations by the likelihood assigned to that value of $Q$ and then adding the four, a general expression relating expected income to planning ratio is obtained:

$$
\begin{equation*}
E(I)=-440.2472+1,619.9614 \rho-796.1664 \rho^{2} \tag{27}
\end{equation*}
$$

This equation is shown in figure 9.
The first derivative of (27) can now be set equal to zero, to solve for $\bar{\rho}:^{19}$

$$
\begin{equation*}
\bar{\rho}=-1,619.9614 /-1592.3328=1.017 \tag{28}
\end{equation*}
$$

That is, the value of $\rho \hat{Q}_{1}$ that would have maximized expected income is 40.7 million pounds. The associated value of $E(I)$, obtained by inserting 1.017 in equation (27), is $\$ 3,838,000$. As indicated in table 10 , the seasonal income that this planning ratio would have implied for the actual volume (46.172) is (according to equation (26)) $\$ 3,883,000$. This amount is approximately midway between the seasonal income actually obtained $(\$ 3,745,000)$ and that attainable with 46.172 ( $\$ 4,046,000$ ).

[^13]Unfortunately, $\bar{\rho}$ is associated only with maximum expected income, not with minimum risk.
Risk With Different Planning Ratios. One measure of risk is the level of the lowest reasonably possible income. It is plausible that there is a maximum tolerable ratio of decrease in this income to increase in expected income. Indeed, management might even specify that the sole criterion for choosing the best planning ratio is to maximize this minimum income. The motivation would be to alleviate disaster if it occurs.

In the present context, the lowest reasonably possible income is the smaller of that associated with $Q=33.828$ and that associated with $Q=46.172$. (Since these volumes correspond to $\pm 1.54$ standard deviations, the odds against a more extreme volume are about seven to one.) As $\rho$ declines from 1.0, the former rises (until $\rho \hat{Q}_{1}$ reaches 33.828), while the latter declines. This is shown in figure 9 , where equations (23) and (26) are graphed. At some value of $\rho$, the two will be equal; at any other value of $\rho$, one or the other would be smaller.

This value of $\rho$ can be estimated by setting equation (23) equal to equation (26) and solving for $\rho$ :

$$
\begin{equation*}
1,312.3154-2,215.545 \rho+872.5 \rho^{2}=0 \tag{29}
\end{equation*}
$$

This implies $\rho=0.941$.
That is, a planning ratio of 0.941 would maximize the minimum likely income. The resulting level of minimum likely income, obtained by inserting 0.941 into equation (23) or (26), would be $\$ 3,620,000$. This contrasts with a minimum likely income of $\$ 3,497,000$ for $\rho=1.017$. There would, of course, be an associated sacrifice in the level of expected income. According to equation (27), $E(I)$ would be only $\$ 3,792,000$ for $\rho=0.941$, compared with $\$ 3,838,000$ with $\rho=1.017$.

Another measure of risk that is useful here is the mean deviation of the probability distribution of outcomes. The mean deviation is equal to two times the sum of the deviations of below-expected incomes from the expected income, each deviation being weighted by its likelihood. The mean deviation indicates the average amount that the possible outcomes differ from the expected outcome. In contrast, the minimum likely income calculated above is the highest level that a prediction of income can take if the odds against a lower outcome are to be at least seven to one; this indicates the "loss" that would result only from one particular unfavorable outcome.
As indicated by the totals in column (15) of table 10, the mean deviation falls steadily as $\rho$ declines from 1.017. The figure for $\rho=1.017$ is $\$ 133,000$; for $1.01, \$ 122,000$; for $1.00, \$ 107,000$; for $0.99, \$ 100,000$. Relative to the associated expected incomes, these amounts are, respectively, 3.5, 3.2, 2.8, and 2.6 per cent.

Extrapolating to lower values of $\rho$ with the aid of equations (23) to (26), we can estimate that the mean deviation would be minimized at about $\rho=$ 0.982 . The relation between the two is indicated in figure 9 .

Conclusions Concerning Uncertainty. Risk, however measured, apparently decreases for a time as $\rho$ declines from unity, but expected income also decreases, conversely, as $\rho$ rises from unity. These conclusions relate to an initially predicted volume that is less than the volume for maximum fresh income. Presumably opposite results would have appeared if $\hat{Q}_{1}>45.2185$.

The conflict between minimizing risk and maximizing expected income would impose a choice between the two if the range of relevant $\rho$ 's was large. The range here, however, appears to be small relative to the uncertainty surrounding $\widehat{Q}_{1}$ itself. It makes little sense to worry whether the $\rho$ component of $\rho \hat{Q}_{1}$ should be 0.98 or 1.02 when the $\hat{Q}_{1}$ component has a standard error of the order of 10 per cent. Given this variability and given the conflict between risk and expected income, it seems reasonable to avoid complications and simply settle for $\rho=1.00$.

The analysis of uncertainty considerations, then, has (for present purposes) served a purely negative function. It has indicated that it is reasonable to approach the problem of intraseasonal allocation as if certainty attached not only to weekly demands, but also to $\widehat{Q}_{1}$.

The weekly quantities and prices that $\rho \hat{Q}_{1}=40.0$ would have implied with the actual volume are shown in figures 6 and 7 by the paths labeled "optimal for $\hat{Q}_{1}=40, Q=46.172$." Compared with the solution in which $\widehat{Q}_{1}=Q=$ 46.172, early quantities are, of course, lower and early prices higher.

In the paths actually chosen by Calavo, however, early quantities are still lower and early prices still higher, although $\hat{Q}_{1}=40$ and $Q=46.172$ were what Calavo had to work with. The result was that Calavo's actual seasonal income was only $\$ 3,745,000$. This is $\$ 90,000$ less than the $\$ 3,835,000$ shown in table 10 for $\rho=1.00$ and $Q=46.172$ - even though the latter income was reduced because replanning occurred only quarterly.
Of course, we have yet to consider the limitations imposed by fruit maturity and storeability.

## With Storage Limitations

It is apparent from figure 6 that, even for an initial prediction of 40 million pounds, actual quantities were too small from October through January and too large from February through July. Why this occurred is evident from the maturity pattern shown in table 6 , in combination with the storage possibilities shown in table 5. In October, November, and December few Fuerte variety avocados are ready to be picked, and most of the Hass and MacArthur fruits have already been sold. By March, on the other hand, the Fuerte crop has become ready, and the choice is between heavy harvesting and deterioration. Nevertheless, there was room for adjustment.

With respect to Fuertes, it should have been possible to increase the quantities sold in December and January-as estimates indicate that other handlers did-and also to increase the quantities held back until May and June.
With respect to Hass and MacArthur, it should have been possible to postpone the start of significant movement from April until the last part of the Fuerte season (June), and also to carry over larger quantities until October
Table 11
ACTUAL VERSUS OPTIMAL MONTHLY QUANTITIES BY VARIETY, 1958-1959

| Month | Variety |  |  |  |  |  |  |  | Total |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fuerte |  | Hass |  | MacArthur |  | Other |  |  |  |
|  | Actual quantities (per cent of 46.172) | Optimal for $\hat{Q}_{1}=$ $Q=46.172$ (per cent of 46.172) | Actual quantities (per cent of 46.172) | Optimal for $\hat{Q}_{1}=$ $Q=46.172$ (per cent of 46.172) | Actual quantities (per cent of 46.172) | $\begin{gathered} \text { Optimal } \\ \text { for } \hat{Q}_{1}= \\ Q=46.172 \\ (\text { per cent of } \\ 46.172 \text { ) } \end{gathered}$ | Actual quantities (per cent of 46.172) | Optimal for $\hat{Q}_{1}=$ $Q=46.172$ (per cent of 46.172) | Actual quantities (per cent of 46.172) | $\begin{gathered} \text { Optimal } \\ \text { for } Q_{1}= \\ Q=46.172 \\ \text { (per cent of } \\ 46.172 \text { ) } \end{gathered}$ |
| October. | 0.1 | ... | 1.7 | 1.7 | 1.0 | 1.7 | 1.0 | 2.0 | 3.8 | 6.4 |
| November. | 1.9 | 1.2 | 0.3 | 1.0 | ... | 0.5 | 1.2 | 3.5 | 3.4 | 6.2 |
| December. | 4.3 | 6.0 | ... | ... | $\ldots$ | ... | 1.5 | 0.8 | 5.8 | 6.8 |
| January . | 6.5 | 8.0 | ... | $\ldots$ | ... | $\ldots$ | 1.7 | 0.7 | 8.2 | 8.7 |
| February. | 10.0 | 9.4 | ... | $\ldots$ | $\ldots$ | ... | 1.0 | 0.7 | 11.0 | 10.1 |
| March . | 12.8 | 12.0 | $\ldots$ | ... | $\ldots$ | $\ldots$ | 1.4 | ... | 14.2 | 12.0 |
| April. | 12.6 | 10.7 | 0.1 | $\ldots$ | $\ldots$ | ... | 0.9 | 0.7 | 13.6 | 11.4 |
| May.. | 8.7 | 9.6 | 0.6 | . | 0.3 | $\ldots$ | 1.4 | 0.8 | 11.0 | 10.4 |
| June. | 3.8 | 4.1 | 2.4 | 2.6 | 1.4 | 0.5 | 1.8 | 1.6 | 9.4 | 8.8 |
| July. | 0.2 | ... | 3.2 | 2.5 | 3.2 | 3.4 | 1.9 | 1.6 | 8.5 | 7.5 |
| August. | ... | ... | 2.9 | 2.8 | 2.3 | 1.3 | 1.0 | 2.4 | 6.2 | 6.5 |
| September... | 0.1 | $\ldots$ | 2.1 | 2.7 | 1.7 | 2.5 | 1.0 | ... | 4.9 | 5.2 |
|  | 61.0 | 61.0 | 13.3 | 13.3 | 9.9 | 9.9 | 15.8 | 15.8 | 100.0 | 100.0 |

Source: Actual quantities based on records of Calavo Growers of California; optimal quantities calculated as described in text.
and November. Full advantage should have been taken of the varieties' ontree storage potential. Larger quantities were needed at the start of the fiscal year, and both Hass and MacArthur could have filled part of the gap.

Additional tonnage in October and November could have been obtained by carrying over from the previous summer larger quantities of Anaheim, Dickenson, Nabal, etc. Advancing the harvest of Bacon and Zutano into November and December also was important. Rincon sales, which heightened the spring peak, should have been postponed as long as storage permitted.

The precise adjustments that should have occurred are, of course, interrelated. If Fuerte quantities in December could not have been increased enough, less Bacon and Zutano should have been advanced from December to November, and more Hass and MacArthur then should have been sold in November instead of October.

One possible solution is indicated in table 11. Optimal monthly quantities are shown which correspond to the weekly quantities that would have been optimal for $\hat{Q}_{1}=Q=46.172$. The varietal breakdown was accomplished by trial and error. Actual 1958-1959 volumes of the principal varieties were reallocated over the season until a distribution was obtained that was consistent with both the target monthly quantities and the maturity and storeability patterns indicated under "Storeability" (p. 720) and "California Maturities" (p. 720). For comparison, table 11 presents estimates, based on members' deliveries, of Calavo's actual sales of the several varieties. Apparently the optimal intraseasonal allocation of the actual volume was feasible.

The path in figure 6 labeled "optimal for $\hat{Q}_{1}=40, Q=46.172$," however, apparently was not feasible. It is true that its quantities in October, November, and December, while larger than the actual quantities, are smaller than those for $\hat{Q}_{1}=Q=46.172$ and therefore easier to match with maturities. But the smaller quantities continue until June, and Fuerte harvesting probably could not have been reduced quite enough in the spring months. Hence the best feasible allocation, given $\hat{Q}_{1}=40$ and $Q=46.172$, lay intermediate between the actual path and the path that was optimal for $\hat{Q}_{1}=40$ and $Q=$ 46.172. Compared with the latter allocation, somewhat greater quantities would have been sold in midseason and somewhat smaller quantities in the final months.

This revision, interestingly, would have increased seasonal income. It would have helped to offset the lowness of the initial prediction. The solution with that prediction, but the actual volume, left heavy quantities for the final quarter. The revision would have transferred quantities from the final quarter, where the last 100 pounds sold each week reduced seasonal income by $\$ 10.91$, to earlier quarters, where $\lambda_{t}$ was positive. As a result, income would have increased from the $\$ 3,835,000$ associated with $\rho \hat{Q}_{1}=40$ and $Q=46.172$ toward the $\$ 4,046,000$ associated with $\rho \hat{Q}_{1}=Q=46.172$.

Hence the extra seasonal income that Calavo could have captured by improved allocation of its 46.172 million pounds, even given $\hat{Q}_{1}=40$, is somewhat more than the $\$ 90,000$ indicated (p. 763)-but of course, still substan-
tially less than the $\$ 300,000$ increase possible in the absence of uncertainty and maturity problems. In round numbers the gain, even with recalculation only quarterly, may be put at $\$ 100,000$, or 3 per cent of actual income, or 0.2 cent per pound. This would have been more than enough to repay the extra trouble of using more refined planning techniques. Presumably a net gain would still remain after subtracting for errors in estimating weekly demands and handling costs, for omitted lagged effects, for any difference in the cost of price guarantees, and for any net changes in fruit value due to altered on-tree storage-and after adding for recalculation every fourth week.

## Recommended Planning Procedures

The foregoing review of the 1958-1959 season has more than historic interest. It indicates that Calavo did remarkably well in 1958-1959. While figures 6 and 7 indicate rather large differences between the actual and the optimum paths for quantity and price, Calavo still realized about 97 per cent of the net income actually available with optimal intraseasonal allocation. Our review also suggests, however, what Calavo might do in future seasons to help assure accurate allocation and to increase returns still further.

The Planning Period. In order to program intraseasonal allocation, it is necessary to decide how many weeks to include in a programming calculation. For convenience, a short planning period is desirable. For accuracy, however, the period should extend substantially into the future, so that quantities for all intervening weeks can be planned simultaneously. Otherwise the planner, in order to decide how much to sell during the planning period chosen and how much to carry over, needs to assign a value to fruit carried over. This value would be arbitrary, and too much or too little might be carried over.

There is a "natural" cutoff date if events are repetitive. In this case it may be reasonable to suppose that conditions during the next cycle will simply duplicate those prevailing currently. Then a zero carry-over value may be assumed, since it would not pay to sacrifice current income for an equal amount of future income. The implication with annual harvests would be use of overlapping planning periods that always terminate 52 weeks from each time of programming, or 104 weeks ahead if an alternate bearing pattern prevails.

Shorter periods, however, may be available. Without distortion of optimal feasible quantities, a planning period can terminate (and zero carry-over value be assumed) at any time that optimal carry-over of currently saleable fruit will be negligibly small. Optimal carry-over will be negligible at some date if thereafter a superabundance of newly matured and perishable fruit will become available, whereas previously, maturities will have held feasible quantities below optimal quantities. Are any such short and nonoverlapping periods available?

The fiscal year-October through September-would not serve, nor would any period that terminates in the months June through November. Optimal carry-over at the end of each of those months will be substantial in order to
augment the small quantities that mature during the fall months. June through December must be included in a single planning period in order to allocate the Hass, MacArthur, Anaheim, Dickenson, and Nabal crops accurately over the summer and into the fall months, and the available Bacon and Zutano fruits over November and December. Similarly, termination could not occur in any of the months January through May without splitting the interval during which discretion can and must be exercised as to the allocation of Fuerte (and new Rincon and Hass) production.

Termination in late December, however, appears tolerable. By January, large quantities of Fuertes are becoming available, and the situation changes to one in which current maturities are at least as large as optimal quantities. As a result, optimal carry-over from December into January should usually be minimal. To be sure, December termination splits the intervals over which Bacon and Zutano fruits become available. No inaccuracy will result, however, because all of these fruits that are ready before January would and should be used to meet November and December quotas.

Since only December termination appears acceptable, we may tentatively settle for nonoverlapping planning periods with a maximum duration of one year: approximately January 1 through December 31. The exact termination date should depend on when new Fuertes are expected to become abundant, and may be adjusted in the recalculations that occur as the season progresses. With this period it will be necessary to estimate Fuerte availability during December as early as the preceding January. This was inescapable. However, it will now be unnecessary to estimate fruit availability beyond the next December.

Implementing Orderly Marketing. The outline below summarizes the steps that Calavo should take in order to implement orderly marketing.
I. Adopt January through December as a tentative planning period.
II. Every fourth week, for example, calculate the optimal feasible quantity for each of the second through fifth weeks following:
A. Calculate the then-current relation of marginal net income $\left(\lambda_{t}\right)$ to total Calavo fresh volume during the remainder of the planning period $\left(Q_{t}\right)$. (This relation corresponds to equation (15b) above.)

1. Obtain the parameters of the latest estimate of the relation between handling costs and weekly quantity. (This relation corresponds to equation (2) above.)
2. Obtain the parameters of the latest estimate of the relation that Calavo average price would have to Calavo quantity in each remaining week of the planning period. (These relations correspond to equations (3) above.)
$a$. Update the generalized function for weekly demand. (This function is given under "Final Equation," p. 738.)
1) Revise this function annually, for example, using more recent data.
2) Alter the constant in this equation by the arithmetic mean of
the differences between actual and predicted Calavo average price over the last four weeks. ${ }^{20}$
$b$. Estimate values of the shift variables in the generalized func-tion-income, non-California quantity, and non-Calavo California quantity-for each remaining week of the planning period.
$c$. For each remaining week of the planning period, insert into the generalized function the number of the week and the latest estimates of the values of the shift variables; calculate the resulting net relation of predicted Calavo average price to Calavo quantity. ${ }^{21}$
3. Decide what weekly interest rate is appropriate, and obtain the weekly ratios that will make income realized in different weeks comparable. (These ratios correspond to equation (8) above.)
4. Perform the calculations indicated by equation (15a) on the demand and cost parameters and interest ratios. The result is the relation of $\lambda_{t}$ to $Q_{t}$.
B. Estimate the amount of Calavo production that will be available during the remainder of the planning period $\left(A_{t}\right)$.
C. Decide how much of the available production should be sold fresh $\left(Q_{t}\right)$ and how much should be processed or abandoned.
5. Determine how much, if at all, returns could be increased by processing or abandoning part of $A_{t}$.
$a$. Estimate the relation of marginal net revenue per pound from processing to quantity processed per week.
$b$. Estimate the relation of marginal picking-plus-hauling costs to quantity harvested per week.
$c$. Using the relation of $\lambda_{t}$ to $Q_{t}$ obtained under A above, calculate the value of $\lambda_{t}$ that would result from selling all of $A_{t}$ fresh.
$d$. Determine whether the value of $\lambda_{t}$ under $1 c$ above exceeds both zero and the largest product of weekly interest ratio times marginal net revenue from processing. If not, Calavo's seasonal income could be increased by processing (if marginal processing revenue exceeds zero) or abandoning (if zero is greater) to the point at which $\lambda_{t}$ equals the larger.
$e$. Determine whether $\lambda_{t}$ also exceeds marginal harvesting costs. If not, members' returns could be increased by abandoning production to the point at which $\lambda_{t}$ equals marginal harvesting costs.
$f$. If processing or abandoning would increase returns, estimate the rate of increase in seasonal income per pound available with respect to volume diverted. (This relation corresponds to equation (21a) above.)

[^14]2. Estimate the rate of increase in independents' prices with respect to Calavo volume diverted. The magnitude involved is indicated by the coefficient of California volume in the latest estimate of annual Calavo demand. (This function is given on page 731 for 1940-1959 data.)
3. Decide how much, if any, of the profitable processing and abandonment mentioned under $1 d$ and $1 e$ above is worth undertaking in view of the estimated increase in independents' returns.
D. Insert the latest estimate of $Q_{t}$ into the equation obtained under A above that relates $\lambda_{t}$ to Calavo fresh volume, and calculate the associated value of $\lambda_{t}$. (That is, assume that the optimum ratio of planned to predicted fresh volume is one.)
E. Obtain for each remaining week of the planning period the relation of optimal quantity to $\lambda_{t}$. (These relations correspond to equations (13) above.)

1. From A above we have the required demand and cost parameters and weekly interest ratios.
2. Perform the calculations indicated by equations (13).
$F$. Insert the value of $\lambda_{t}$ obtained under D above into each equation for optimal weekly quantity and calculate the indicated optimal allocation of $Q_{t}$ by weeks.
G. Estimate how, if at all, the optimal allocation should be modified to adapt to maturity and storage limitations; compare alternatives by calculating the seasonal income that each would imply. The quantities that result for the next four weeks represent the estimated optimal feasible quantities for the immediate future. The remaining quantities will be replanned next month.
III. Regulate harvesting to conform to the quantity targets just mentioned. (If in doubt, regulate harvesting as was indicated appropriate for 1958-1959, page 763.)
IV. Adjust selling prices to the highest levels at which approximately the target quantities can be sold.
V. Be willing to depart from these target quantities and prices, but only when convincing reasons exist for doing so.
VI. Experiment with replanning intervals other than four weeks. Recalculation may yield different quantity targets any time that a change has occurred in total fresh volume, in maturity or storage limitations, or in the parameters of the weekly demand and handling cost functions. Settle on a fixed or variable interval such that more frequent recalculation apparently would increase seasonal income less than it would increase planning costs, and less frequent recalculation apparently would decrease income more than it would decrease planning costs.
VII. Analyze the relation of expected income and risk to planning ratio (see "Uncertain Total Volume," p. 753) for seasons after 1958-1959, to check whether it still appears that planned fresh volume should equal predicted fresh volume.
VIII. Arrange for continuing performance of the above calculations-by an employee trained in econometrics, by contract with other concerns, by hiring a consultant, etc.
IX. Arrange for research into the relation of fruit grade, size, weight, and weather-loss to on-tree holding time. The foregoing analysis assumes that such changes offset each other in value terms. If research indicates otherwise, the calculations should be modified. Similarly, further study is warranted of interdependence among weeks stemming from price guarantees and from the presence of lagged effects of previous sales.
X. Arrange for the study of weekly demand by geographic areas and of optimum zone differentials. The foregoing analysis uses an aggregated weekly demand and therefore presupposes continuation of the existing pattern of geographic pricing. To equalize marginal net income also by areas could further enhance income.
XI. Consider whether variety recommendations to members should be revised because the maturity and storeability of certain varieties imply that a larger proportion of those varieties would facilitate or impede orderly marketing.
XII. Consider whether existing procedures for determining individual members' returns would now provide the best balance among equity, incentives, pooling costs, and risk spreading.
XIII. Periodically review the adequacy of all the above-from the planning period to pooling categories.

## ACKNOWLEDGMENTS

The author is indebted to Professors Harold Carter, Jerry Foytik, Gordon King, and George Kuznets, each of whom helped to clarify essential points of methodology, and to Professors Foytik and Peter Verdoorn, who reviewed an earlier version and made many incisive comments. Substantial contributions were also made by Mr. Y. H. Mo and Mr. C. Turco, research assistants, and by Mrs. M. Vaage, head of the Department's statistical pool. Much of the descriptive material was supplied by officials of Calavo Growers of Cali-fornia-particularly Mr. D. Freistadt and Mrs. M. E. Lawson.

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[^15]CALAVO SALES OF CALIFORNIA AVOCADOS（THOUSAND POUNDS）FOR MID－MONTH WEEKS，1940－1959

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Source：Records of Calavo Growers of California．


[^16]Appendix Table 4

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Sourcm：Based on records of Calavo Growers of California．For interpretation，see page 717.

| Month | Percentage of hypothetical revenues with all Calavo grade |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | $\begin{gathered} 1939- \\ 40 \end{gathered}$ | ${ }_{41}^{1940}$ | $\underset{42}{1941-}$ | ${ }_{43}^{1942-}$ | $\underset{44}{1943-}$ | ${ }_{45}^{1944-}$ | $\begin{gathered} 1945- \\ 46 \end{gathered}$ | $\begin{gathered} 1946- \\ 47 \end{gathered}$ | $\stackrel{1947-}{48}$ | ${ }_{49}^{1948-}$ | $\begin{gathered} 1949- \\ 50 \end{gathered}$ | $\begin{gathered} 1950- \\ 51 \end{gathered}$ | $\begin{gathered} 1951- \\ 52 \end{gathered}$ | $\stackrel{1952-}{53}$ | $\underset{54}{1953-}$ | $\begin{gathered} 1954- \\ 55 \end{gathered}$ | $\begin{gathered} 1955- \\ 56 \end{gathered}$ | $\begin{gathered} 1956- \\ 57 \end{gathered}$ | $\begin{gathered} 1957- \\ 58 \end{gathered}$ | $\begin{gathered} 1958- \\ 59 \end{gathered}$ |
| October. | 79.6 | 79.6 | 81.7 | 74.7 | 76.5 | 69.8 | 72.1 | 74.7 | 87.1 | 75.9 | 76.2 | 76.0 | 71.0 | 76.8 | 78.8 | 76.5 | 83.3 | 88.6 | 87.4 | 86.1 |
| November. | 92.1 | 93.1 | 92.3 | 91.0 | 90.3 | 88.9 | 91.4 | 91.8 | 93.2 | 90.7 | 90.7 | 90.9 | 81.7 | 83.7 | 91.2 | 86.1 | 75.2 | 82.5 | 90.1 | 86.1 |
| December. | 97.4 | 94.0 | 95.2 | 94.0 | 93.5 | 93.4 | 94.6 | 93.2 | 92.7 | 94.9 | 89.8 | 92.8 | 88.2 | 90.6 | 90.0 | 93.1 | 88.1 | 85.3 | 90.3 | 89.1 |
| January. | 96.4 | 94.0 | 94.9 | 92.5 | 92.9 | 93.9 | 91.6 | 91.7 | 91.9 | 91.7 | 91.2 | 91.6 | 86.7 | 91.6 | 92.1 | 93.0 | 93.6 | 87.5 | 89.5 | 89.4 |
| February. | 95.2 | 94.4 | 94.6 | 91.2 | 92.3 | 94.6 | 92.8 | 91.4 | 90.2 | 93.0 | 91.8 | 92.4 | 88.2 | 90.3 | 92.4 | 93.8 | 95.1 | 89.1 | 91.1 | 91.4 |
| March . | 96.1 | 93.0 | 94.2 | 92.4 | 91.6 | 94.5 | 91.6 | 91.5 | 92.2 | 94.3 | 91.5 | 91.4 | 87.8 | 87.0 | 90.4 | 92.0 | 95.1 | 92.2 | 93.0 | 90.0 |
| April. | 94.7 | 90.0 | 94.1 | 92.4 | 90.1 | 92.8 | 91.8 | 90.6 | 92.4 | 91.9 | 90.7 | 92.2 | 87.5 | 88.6 | 88.7 | 92.0 | 94.8 | 93.6 | 95.6 | 93.2 |
| May. | 91.6 | 88.4 | 91.6 | 91.1 | 90.0 | 90.1 | 91.8 | 89.3 | 90.3 | 90.6 | 88.4 | 90.2 | 87.4 | 89.3 | 88.4 | 90.1 | 93.1 | 92.6 | 94.3 | 94.4 |
| June. | 91.3 | 89.1 | 87.1 | 83.9 | 87.9 | 85.1 | 87.0 | 80.1 | 85.8 | 90.0 | 83.4 | 85.0 | 86.5 | 87.1 | 85.2 | 90.8 | 88.2 | 93.3 | 94.0 | 93.5 |
| July . | 91.6 | 90.8 | 88.0 | 79.6 | 86.7 | 78.4 | 84.8 | 79.5 | 82.9 | 82.1 | 80.3 | 80.3 | 80.5 | 85.4 | 80.9 | 87.8 | 88.9 | 88.3 | 93.0 | 92.9 |
| August. | 82.5 | 89.1 | 88.9 | 77.8 | 87.9 | 80.1 | 84.8 | 79.7 | 79.6 | 78.1 | 77.4 | 79.5 | 79.5 | 83.9 | 80.9 | 88.6 | 89.2 | 84.4 | 92.6 | 93.5 |
| September | 73.4 | 83.8 | 82.1 | 74.2 | 82.2 | 79.4 | 74.8 | 80.5 | 81.2 | 77.5 | 78.7 | 73.0 | 78.6 | 81.0 | 75.5 | 84.4 | 91.6 | 79.9 | 88.8 | 91.2 |

ESTIMATED SALES OF FLORIDA AVOCADOS (THOUSAND POUNDS) FOR MID-MONTH WEEKS, 1940-1959

Source: Annual data given in table 1 times weekly percentages derived from reports of the Avocado Administrative Committee (1955-1960), from unpublished data supplied
to the author by the Florida State Marketing Bureau on 1951-1958 interstate truck shipments, and from extrapolations.

Source: Annual data given in table 1 times weekly percentages estimated from monthly data contained in U. S. Department of Commerce (1940, 1941, 1944-1960) and in an
unpublished special report from the Bureau of the Census to the author pertaining to imports during 1938, 1942, and 1943.
Appendix Table 8
SEASONALLY ADJUSTED ANNUAL RATE OF NONAGRICULTURAL PERSONAL INCOME (BILLION DOLLARS) BY MONTHS, 1940-1959

| Month | Year |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 1939- \\ 40 \end{gathered}$ | $\begin{gathered} 1940- \\ 41 \end{gathered}$ | $\begin{gathered} 1941- \\ 42 \end{gathered}$ | $\begin{gathered} 1942- \\ 43 \end{gathered}$ | $\begin{gathered} 1943- \\ 44 \end{gathered}$ | $\begin{gathered} 1944- \\ 45 \end{gathered}$ | $\begin{gathered} 1945- \\ 46 \end{gathered}$ | $\begin{gathered} 1946- \\ 47 \end{gathered}$ | $\begin{gathered} 1947- \\ 48 \end{gathered}$ | $\begin{gathered} 1948- \\ 49 \end{gathered}$ | $\begin{gathered} 1949- \\ 50 \end{gathered}$ | $\begin{gathered} 1950- \\ 51 \end{gathered}$ | $\begin{gathered} 1951- \\ 52 \end{gathered}$ | 1952- $53$ | $\begin{gathered} 1953- \\ 54 \end{gathered}$ | 1954- <br> 55 | $\begin{gathered} 1955- \\ 56 \end{gathered}$ | 1956- $57$ | $\begin{gathered} 1957- \\ 58 \end{gathered}$ | $\begin{gathered} 1958- \\ 59 \end{gathered}$ |
| October. | 68.8 | 74.7 | 93.0 | 121.0 | 141.9 | 154.3 | 152.9 | 164.9 | 177.1 | 193.4 | 189.5 | 218.7 | 242.2 | 261.4 | 273.9 | 276.2 | 301.9 | 324.0 | 337.9 | 346.3 |
| Novembe | 69.6 | 75.8 | 93.9 | 123.7 | 144.3 | 155.1 | 154.3 | 165.5 | 178.3 | 193.7 | 191.4 | 220.0 | 243.8 | 263.4 | 272.6 | 278.9 | 304.2 | 325.7 | 337.8 | 349.6 |
| December | 70.1 | 78.1 | 97.3 | 125.8 | 145.0 | 156.1 | 153.8 | 167.7 | 180.4 | 193.9 | 195.1 | 229.4 | 245.5 | 265.2 | 272.3 | 281.4 | 308.0 | 327.3 | 335.7 | 348.8 |
| January | 70.5 | 79.5 | 98.6 | 129.1 | 146.6 | 158.1 | 155.6 | 168.3 | 182.1 | 192.6 | 199.2 | 227.0 | 246.2 | 266.1 | 270.0 | 281.3 | 307.3 | 327.9 | 335.6 | 351.6 |
| February | 70.4 | 80.0 | 99.6 | 131.7 | 149.0 | 158.4 | 153.1 | 167.8 | 182.7 | 192.2 | 203.3 | 229.5 | 248.4 | 268.5 | 270.7 | 282.9 | 308.6 | 330.3 | 333.1 | 353.8 |
| March | 70.3 | 81.8 | 102.0 | 133.5 | 149.6 | 159.4 | 157.5 | 168.4 | 185.2 | 191.9 | 208.5 | 231.2 | 249.1 | 270.5 | 270.5 | 285.9 | 311.4 | 331.5 | 333.6 | 358.5 |
| April | 70.1 | 82.8 | 104.4 | 134.8 | 148.9 | 158.1 | 158.6 | 167.6 | 185.4 | 193.2 | 204.3 | 233.5 | 249.1 | 271.1 | 271.5 | 289.5 | 314.9 | 333.1 | 335.1 | 362.7 |
| May. | 71.2 | 85.4 | 106.5 | 135.2 | 150.1 | 158.6 | 159.8 | 168.7 | 186.9 | 192.8 | 204.2 | 235.0 | 251.1 | 271.9 | 272.0 | 292.5 | 315.8 | 335.8 | 337.1 | 365.3 |
| June. | 71.5 | 88.0 | 109.9 | 137.1 | 151.4 | 160.6 | 161.1 | 170.8 | 189.0 | 191.8 | 207.1 | 238.3 | 251.8 | 273.0 | 272.7 | 294.1 | 318.1 | 337.9 | 339.9 | 367.8 |
| July | 72.0 | 89.4 | 113.0 | 138.7 | 152.5 | 160.6 | 161.3 | 170.7 | 191.0 | 191.1 | 209.1 | 237.4 | 249.7 | 273.7 | 272.8 | 298.2 | 316.9 | 338.5 | 345.7 | 368.2 |
| August. | 73.0 | 91.3 | 115.5 | 139.7 | 152.8 | 156.1 | 163.5 | 171.3 | 192.9 | 191.4 | 213.3 | 239.0 | 256.3 | 273.7 | 273.0 | 297.8 | 320.4 | 339.0 | 344.5 | 366.3 |
| September. | 73.6 | 92.2 | 117.8 | 140.3 | 152.7 | 150.3 | 165.1 | 184.4 | 193.6 | 191.9 | 217.8 | 240.0 | 259.0 | 273.0 | 274.1 | 300.1 | 322.2 | 338.5 | 346.1 | 367.5 |
| Average. | 70.9 | 83.2 | 104.3 | 132.6 | 148.7 | 157.1 | 158.0 | 169.7 | 185.4 | 192.5 | 203.6 | 231.6 | 249.4 | 269.3 | 272.2 | 288.2 | 312.5 | 332.6 | 338.5 | 358.9 |

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[^0]:    ${ }^{1}$ Submitted for publication August 4, 1961.
    ${ }^{2}$ Assistant Professor of Agricultural Economics and Assistant Agricultural Economist in the Experiment Station and on the Giannini Foundation, Davis.

[^1]:    ${ }^{3}$ See "Literature Cited" for citations referred to in the text by author and date.
    ${ }^{4}$ Garcia, Crews, and Roberts, wholesale produce dealers, Havana, Cuba, personal correspondence, May 5, 1958.

[^2]:    ${ }^{5}$ Robert W. Hodgson, personal correspondence, April 9, 1959.

[^3]:    ${ }^{6}$ This average total cost function was obtained in four steps:
    (1) Assuming an exponential form, annual cost data for the last four years yielded the equation $u=104 Q^{-0.7}$, relating cents per pound to million pounds per season.
    (2) It was assumed that unit costs are related to weekly tonnage in the same way that they are related to annual tonnage, that is, it was assumed that weekly unit costs are given by the equation $u=104(s \hat{q})^{-0.7}$, where $\hat{q}$ represents thousand pounds per week and $s$ is an unknown scalar. To find $s$, the observed annual unit costs were regarded as weighted averages of unknown weekly unit costs. This implies an equation which, knowing weekly tonnages can be solved for $s$ (and $s^{-0.7}$ ) for each season: known annual unit cost $=$

    $$
    \sum_{w=1}^{52}\left[\hat{q}_{w}(104)\left(s \hat{q}_{w}\right)^{-0.7}\right] / \sum_{w=1}^{52} \hat{q}_{w}
    $$

    Values of $s^{-0.7}$ were computed for the last three seasons (using only the middle week of each month). They were 7.34, 8.92, and 8.16 (implying values for $s$, when $q$ and $Q$ are both expressed in million pounds, of 57.1, 43.1, and 49.0). The average, 8.14, times 104 equals 850 . Hence we have $u=850 \hat{q}^{-0.7}$.
    (3) This function was approximated by one with a form that will permit solution of orderly marketing problems: $u=\alpha q^{-1}+3.0-0.5 q$, where $q$ is now expressed in million pounds for convenience, and $\alpha$ is an unknown constant.
    (4) $\alpha$ was assigned the value which makes the function predict actual handling costs in 1958-1959 ( $\$ 2,962,000$ ) perfectly from the actual weekly quantities. The result is given in the text.
    ${ }^{7}$ Marginal costs are shown to decrease with quantity. This result is not justified by, but rather is a consequence of, having fitted an exponential function to data that combine fixed and variable costs. (If marginal costs were constant, the exponent -0.7 would merely imply that fixed costs historically averaged 70 per cent of total costs.) Instead, the justification is that two million pounds per 40-hour week is the designed pack-out, and it is plausible that marginal costs of packing plus selling decrease slightly up to that level.
    ${ }^{8}$ Field-plus-packing costs alone can be estimated with more confidence, since figures for this category represent costs of handling only California avocados, not varying amounts of companion lines as well. Using the procedure and symbols discussed under (1) and (2) in footnote 6 , except that we now have quantity packed not quantity sold, I obtained $u=$ $12 Q^{-0.3}=30 \hat{q}^{-0.3}$. Values of $s^{-0.3}$ here were 2.47, 2.53, and 2.49, the average of 2.5 implying an $s$, dimensionally adjusted, of 47.2 . The annual function here fits the data almost perfectly-much closer than does the function in the text.

[^4]:    *"General office expense" and "other marketing expenses" minus gross profit on companion lines.

    Source: Distributors' share is an estimate based on interviews. Other figures are based on data in annual reports of Calavo Growers of California.

[^5]:    ${ }^{9}$ This formulation is less question-begging than the reverse relation-the average prices that would be received if various alternative quantities were offered-since the average price for any quantity will vary with the transaction system (ordinary auction, Dutch auction, asking prices plus bargaining, etc.), with collusion, and so forth. As a result, with the latter formulation the particular "values" of shift variables must include a specified transaction system, degree of collusion, and so forth. The function then becomes less general, reflecting not only buyers' responses but also a particular market structure and conduct.
    ${ }^{10}$ Thus the typical situation at an ordinary auction, with many intramarginal buyers, seems likely to be one in which at least a few of the buyers will be misinformed, anxious, or naive and, as a result, will bid early prices above the ex ante level long enough to shift up the price expectations of other intramarginal buyers, who then will keep prices above the ex ante level until the last round. No empirical test has yet been made of this hypothesis.

[^6]:    ${ }^{11}$ Values adjusted for degrees of freedom are: $\bar{R}^{2}=0.806, \bar{S}_{1.23}=3.5$.
    ${ }^{12}$ If income is multiplied by 1.053 , log income increases by the logarithm of 1.053 , or $0.0224 ; 0.0224$ times the coefficient of $\log$ income, 44.6 , equals 1 cent.

    The actual rate of growth in nonagricultural income between successive crop years averaged, for 1947-1948 through 1958-1959, 6.24 per cent. This corresponds to an upward shift in demand of 1.2 cents per pound per year.
    ${ }^{13}$ Price flexibility with respect to volume is $[\partial P / \partial C][C / P]=-0.327 C / P$. At the centroid, the latter equals -0.327 [44.9/21.2] $=-0.69$.

    Price flexibility with respect to income is $[\partial P / \partial Y][Y / P]=\left[44.6 \log _{10} e / Y\right][Y / P]=$ $19.37 / P$. At the centroid, the latter equals $19.37 / 21.2=0.91$.

    More common measures of proportionate responsiveness are the price-elasticity of demand and the income-elasticity of demand. Price elasticity is $[\partial C / \partial P][P / C]$. Income elasticity is $[\partial C / \partial Y][Y / C]$. These measures apply where volume can be regarded as an endogenous variable. In the present context a value for $\partial C / \partial P$ or for $\partial C / \partial Y$ would be, if not meaningless, at least statistically unreliable, since volume appears in fact to be substantially predetermined. For purposes of comparison, however, it may be worthwhile to indicate what elasticities "correspond" to the flexibilities presented in the text.
    A value for $\partial C / \partial P$ in the present context could plausibly be obtained in two ways. One is by inverting and differentiating the regression equation given in the text, which

[^7]:    was fitted with price as the dependent variable. The other is by finding the value of $\partial C / \partial P$ that would apply to a new regression equation, fitted with volume as the dependent variable. With less-than-perfect correlation, the two values will differ. The latter value seems the more objectionable; the validity and reliability of an equation fitted with a predetermined variable taken as endogenous is doubtful. Hence, the corresponding price-elasticity at the centroid may be taken as $[-1 / 0.327][21.2 / 44.9]=-1.44$, the reciprocal of the price flexibility.

    Similarly, to obtain $\partial C / \partial Y$, it is probably least objectionable to invert and differentiate the equation in the text, obtaining (44.6/0.327) $\log _{10} e / Y=59.24 / Y$. Income elasticity then equals $59.24 / C$, which at the centroid equals 1.32 .

[^8]:    ${ }^{14}$ The result was $\hat{p}=-69.8-28.6 c+1.46 c w-0.0290 c w^{2}-6.63 n+46.7 \log y$. $(t=8.71)(t=6.43) \quad(t=7.17) \quad(t=8.07) \quad(t=23.12)$

[^9]:    $\stackrel{*}{\dagger}$ Columns (4) times (7) divided by (5).

[^10]:    ${ }^{15}$ This amount represents the sum of actual weekly quantities times actual weekly average f.o.b. prices. The Annual Report shows sales minus transportation costs lof $\$ 6,520,000$. The difference may reflect faulty averaging and/or accounting in the Report for a delivery year instead of the 52 -week sales year. If the reported figure were used, the result would be an even greater advantage for the optimal allocation.
    ${ }^{16}$ This amount equals the reported figure, since the handling cost function was chosen with this result in mind (see "Handling Costs," p. 725).

[^11]:    ${ }^{17}$ Suppose the objective were to maximize, not $I$, but $I$ minus picking and hauling costs-that is, members' on-tree returns. Then $\lambda$ would in effect be set equal, not to zero, but to marginal harvesting costs. If these are considered to be a constant 1 cent per pound, the answer would be $Q=43.5$.

[^12]:    ${ }^{18}$ To neglect risk and consider only $E(I)$ would imply the dubious assumption that every $\$ 10,000$ of uncaptured attainable income is equivalent to Calavo regardless of the level of income that is to be decreased by $\$ 10,000$.

[^13]:    ${ }^{19}$ The value of $\rho$ that maximizes $E(I)$ also maximizes the expected proximity of actual to attainable income, or $E(I-\bar{I})$. To estimate $E(I-\bar{I})$ (before multiplying by the likelihoods and summing) subtract 367.9 from equation (23), 386.4 from equation (24), 403.6 from (25), and 404.6 from (26). After discounting and summing, the value for $E(I-\bar{I})$ would equal equation (27) minus a constant, and would therefore reach a maximum at the same value of $\rho$.

    Maximizing $E(I-\bar{I})$ was the criterion adopted by Mundlak (1956). In certain respects this criterion leads to a more manageable analysis. Unfortunately, Mundlak's excellent treatment of the subject came to the author's attention only after the present study was substantially completed.

    Mundlak recognized that the optimal value of planned volume is not necessarily equal to the expected value of actual volume. He concluded (p. 1499) that "the optimal forecast is obtained by adding a constant to" $\hat{Q}_{1}$.

[^14]:    ${ }^{20}$ This adjustment is appealing intuitively. It is also justified by the presence of serial correlation in the residuals of the generalized function, and by the prospect of positively correlated errors in estimates of the shift variables.
    ${ }^{21}$ An alternative to steps $b$ and $c$ is to obtain the implied intercepts of the weekly demand functions during the previous season-as was done in equations (5) above-and then to apply the adjustment indicated in step $a(2)$.

[^15]:    Source：Records of Calavo Growers of California．

[^16]:    Source：Records of Calavo Growers of California．

