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Growth Equations and Curves for Citrus Trees

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Percival Allen, J. H. Gaddum, and S. C. Pearce writing in Nature in 1945, all have emphasized the advantages of using the simple and powerful methods afforded by logarithmic transformations in analyzing nonnormal distributions, although it had been amply demonstrated in the relative growth of animal parts by Huxley (1932). We have undertaken to illustrate graphically the use of logarithm and power transformations for growth models of trees in orchards and tree organs. Various parameters based on literature, either age or size dependent, are described by power functions, log-log linear curves of the type $y = bx^k$, or semi-log linear curves, exponential functions where $y = ae^{bx}$. Tree height or trunk diameter versus tree age, tree-leaf surface area or number on the tree versus tree age, leaf area versus length, or leaf area versus width are linear log-log functions. It is shown that the first pair of parameters are not normally distributed; latter pairs were demonstrated to be normal. Fruit yield was a nonlinear logarithmic function of tree age and their annual size-frequency distributions were not normal, except for infinitely large populations. Individual fruit size is a linear function of log fruit age, but only the log-log relations are linear for fruit dimensions (diameter, volume, and mass) versus packing number. Log branch fresh weight, leaf and fruit fresh weight are linear functions of log tree age, as are logs of branch dry weight, of branch diameter, of number of branches and surface areas and of volumes. High positive correlations between woody organ ages and dry organ densities invalidate the Rashevsky theoretical growth equation. Insertion of new density terms satisfy validity requirements. Frequency distribution of branch-diameter per tree is a linear log-log function. It is postulated that the linearity of the log-log and semi-log dimensional relationships in plant growth result from similar physical and chemical relationships underlying growth as outlined by kinetic theory.

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Growth Equations and Curves for Citrus Trees[1]

INTRODUCTION

RESEARCH INVESTIGATIONS DEALING WITH WOODY PLANTS, especially trees, are more difficult, because of their perennial character, than research studies of herbaceous plants. These difficulties arise primarily because of problems associated with the scientific method. A plant's natural activities such as transpiration, respiration, photosynthesis, growth, reproduction, and heat transfer, or the modification of these activities by man as a result of irrigating, controlling plant competitors (such as plant diseases, insects, and others), spraying, dusting, gassing, pruning, etc. are expressed on a unit-area, wet-weight or dry-weight basis. They could just as readily be expressed on a leaf basis or tree basis if some invariant relationships could be established. The size or age of such structures as trees generally precludes the acquisition of weight or area measurements. Rarely is the structure sacrificed, because size, age, and value are strongly linked. But data for a single tree, especially a young one, is of no help. A series of ages and a corresponding series of size data are necessary, and each set of data usually requires the sacrifice of at least one valuable tree. However, the amount of work and time required may be so great, even for a single large old tree, that it seems unwise to undertake such a study. The cost of using a series of whole trees is prohibitive because the cost of trees and labor is so great. Resort, therefore, is made to a sampling procedure with a statistical design capable of describing the variability within an age group. Usually, such analyses have not included a sufficient number of age groups to describe a curve of a given parameter versus age. Thus, it has often been impossible to generalize on the rate of change of a given dimension with age.

Two approaches to the solution of the problem are possible. The first is automation of the processes required to obtain the desired dimensions. Packinghouse methods for citrus fruit is a good example. Fruit are automatically sized on an equatorial diameter basis, sorted, and counted. Leaf picking, sizing (length, width, area, and thickness), sorting and counting on a size basis could be automated. Wet weight and percentage of moisture might also be automatically determined. No such devices have so far been developed.

Although various parameters of fruit and leaves have yielded or will yield readily to automation, the outlook is much less promising for root systems or parts of the woody-frame of the tree above ground. Both roots and branches are multipointed, irregular in shape, and have an extreme range in number-per-size class. Thus, at present and possibly for some time to come, the parameters of root and branch systems will have to be determined by traditional methods.

The second approach to the problem is to establish growth equations and curves for a particular variety grown

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under specified climatic conditions and any other variable of the many that strongly affect growth (Monselise and Turrell, 1959; Turrell, *et al.,* 1964; Parker, *et al.,* 1940). This approach, although convenient, is not greatly important where automation can be applied but it is most critical where automation can not.

The present paper reports on growth curves, and equations of the curves for various parameters of above-ground parts of citrus trees in which the exponent is invariable (power function) or variable (exponential function). In the former case, the base is variable; in the latter, invariable. The data used has largely been drawn from published works.

In utilizing any estimate so obtained, it is important to know whether the mean is drawn from a population with a normal frequency distribution, a skewed distribution, or one of several other distributions such as log-log. Frequency distributions for leaves, stems, and fruit have therefore been included in this report based on original data.

TREE HEIGHT AND TRUNK DIAMETER

Tree height and trunk diameter were determined on four California Valencia orange trees of different ages grown

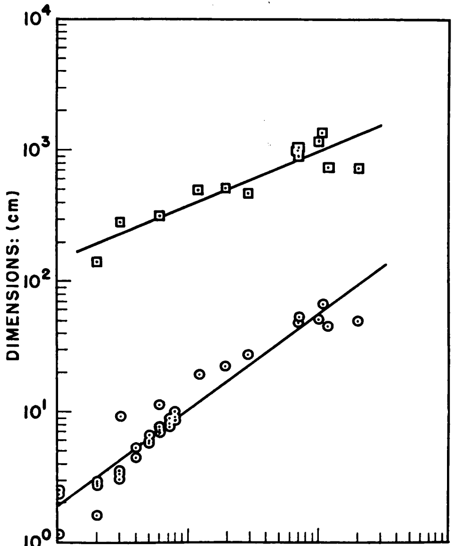


Fig. 1. Linear growth curves of log of tree height (upper curve) and log of trunk diameter (lower curve) vs. log of tree age. Many varieties of citrus in world-wide locations. Data from Webber and Batchelor (1943).

from buds of the same "Campbell" clone and on the same sweet-orange rootstock. The four trees, ages 3, 6, 12 and 29 years, were 2.90, 3.20, 5.03 and 4.72 meters tall, respectively. They had crown circumferences of 7.26, 7.32, 11.02 and 15.70 meters, respectively, and trunk diameters of 9.6, 11.3, 19.2 and 27.2 centimeters, respectively. When plotted on arithmetic graph paper these dimensions plot as "S-shaped" curves against tree age. But when plotted on graph paper having logarithmic scales for both axes, nearly straight lines were formed. Mutual shading of the trees in the orchard as they age seems to be responsible for the logarithmic effect of tree age (Turrell, 1961). These are very general responses, as shown in figure 1 where the growth of commercial citrus trees of many kinds from all over the world (Webber and Batchelor, 1943) are plotted.

The frequency distribution curves of trunk diameters of citrus trees growing in the orchard may be far from normal. Analyses of trunk diameters of two hundred Washington Navel orange trees for skewness and kurtosis at approximately five and ten year intervals showed that only at planting time were trunk diameters normally distributed. At all subsequent samplings the trunk diameter frequency distribution curves were highly significantly skewed and kurtotic (0.1 percent level) as shown in figures 2 and 3.

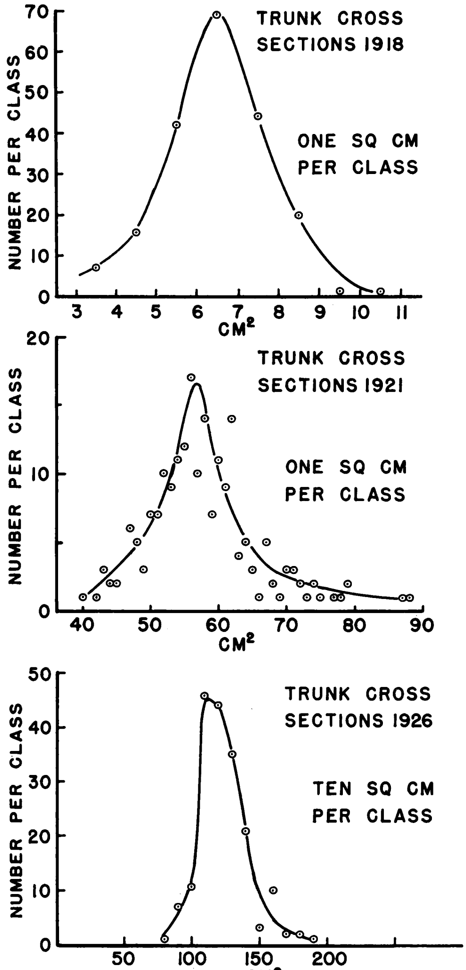


Fig. 2. Frequency distributions of trunk cross sectional areas of two hundred Washington Navel orange trees on sweet-orange rootstock from date of planting to pulling. Curves shown for three year and five year intervals, with 1 cm² per class for 1918 and 1921 and 10 cm² per class in 1926. Note the tendency to proceed from normal ("t" test for kurtosis and skewness, non significant, in 1918) to kurtotic and skew distributions at all subsequent samplings ("t" tests for kurtosis and skewness are all highly significant).

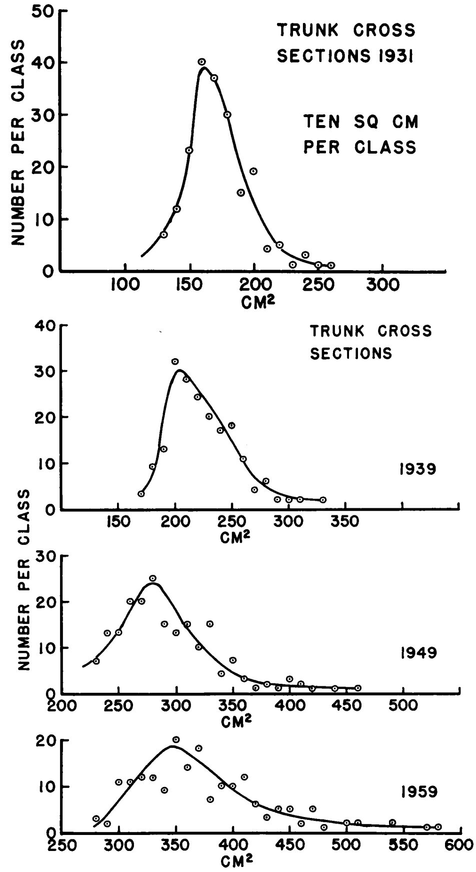


Fig. 3. Continuation of samplings shown in figure 2 for five and ten year intervals. Note the increasing degree of kurtosis with age. The "t" tests showed that the curves all varied with respect to skewness and kurtosis highly significantly from normal. Trees were grown in the Department of Horticulture plots of the University of California Citrus Experiment Station, Riverside.

LEAF SURFACE AREA AND LEAF NUMBER

Growth of leaf area of California Valencia orange trees followed the same patterns as growth of trunk diameter and tree height referred to above. The youngest tree sampled (three-year-old) had 16,419 leaves, a six-year-old 37,257, a 12-year-old 92,708, and the oldest, a 29-year-old tree, 172,613 leaves. The total leaf area of each of these trees was 34, 59, 146, and 203 square meters respectively (Turrell, 1961). Plotted against tree age on log-log paper, both the leaf number per tree and the total leaf area per tree gave straight lines. The straight line representing increasing leaf number (N) for trees of increasing age, α, in years is given by equation 1.

$$\log N = 3.613 + 1.249 \log \alpha \qquad (1)$$

The leaf area, A, plotted in figure 4 may be obtained in square meters from equation 2.

$$\log A = 0.994 + 1.068 \log \alpha \qquad (2)$$

On each of the four trees the leaves were "normally distributed" according to blade length (Turrell, 1961). In each of four commercial varieties, as the individual leaves increased in length, l, they increased in area (A') by the same log-log law. The average area for a Valencia orange leaf of a given length is represented by the following equation:

$$\log A' = -0.423 + 1.923 \log l \qquad (3)$$

where A' is leaf blade area in square centimeters and l is leaf blade length in centimeters (Turrell, 1961). Also, for Valencia oranges grown in solution culture, the size relationships (Chapman and Parker, 1942), are unchanged. Calculations have also shown that in the latter work, the log of leaf number and log of leaf area are linear functions of log of tree age.

FRUIT YIELD AND FRUIT SIZE

A number of studies have dealt with fruit yield and fruit size. Specifically increasing either one or both of these

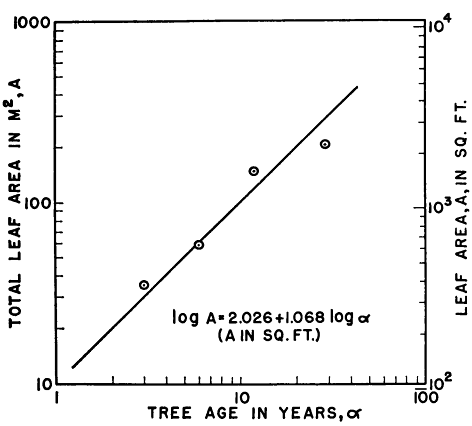


Fig. 4. Linear growth curve for log of total leaf area per tree plotted against log of tree age of Valencia orange trees. Data from Turrell (1961).

two parameters of tree growth has been the primary concern of citrus experiment stations for many years.

Savage (1960) compiled tables for low, average, and high yielding trees, of different ages, for the five types of citrus fruit grown in Florida. The data cannot be plotted in straight lines on arithmetic, semi-log or log-log graph paper because the cumulative growth curve has a drawn out ʃ-shape. These curves can be fitted by polynomials, and thus, equations of increasing yields with tree age will best be developed by using electronic computers. But semi-log linear curves can be fitted to each of the three stages: 1, early (lower limb of the ʃ); 2, middle (the rapid-rising central portion); and 3, late growth stages (upper limb of the ʃ). For example, if the yields of middle-aged grapefruit trees are plotted on semi-log paper, the straight line of increase with age doesn't make a sharp break until the trees are between 25 and 30 years old. From then on, yield shows little further increase and the curve becomes asymptotic. Simplified equations for grapefruit yield can be written that apply from start of maturity up to the inflection point of old age; e.g., if Y = yield in boxes per tree, and a = tree age in years, then for:

Low yield trees Y =

$$6.34 \log a - 4.50, \qquad (4)$$

Average yield trees Y =

$$8.43 \log a - 4.36, \qquad (5)$$

High yield trees Y =

$$13.46 \log a - 8.32. \qquad (6)$$

Studies on Washington Navel orange yields at the Citrus Research Center, and Agricultural Experiment Station, University of California, Riverside, have shown that size-distribution of fruit in any one year is far from a normal distribution (figures 5 and 6). However, when all of the grapefruit size data accumulated by the Desert Grapefruit Marketing Program (Showers, 1943–50; Grunow, 1951–61) were plotted, a normal distribution curve for Arizona grapefruit (figure 7) resulted.

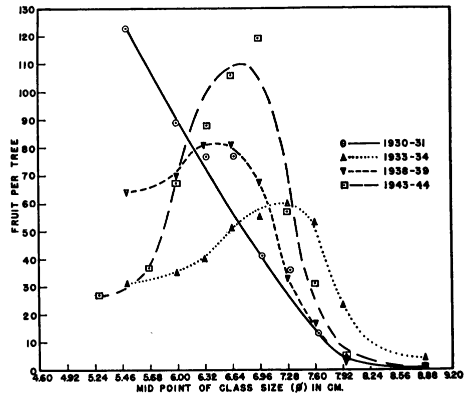
The growth in diameter (ϕ) of citrus fruit (oranges, grapefruit and lemons) versus days from set (a') are curvilinear on rectilinear coordinate paper but can be represented by equations of the exponential type:

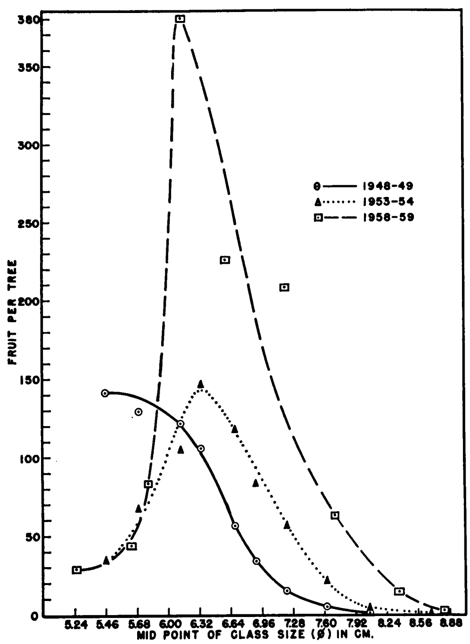
$$\phi = a + b \log a' \qquad (7)$$

For example, the equation for the increase in diameter (cm) of lemons with age based on lemon growth data by Bartholomew (1923) is:

$$\phi = 6.403 \log a' - 10.095 \qquad (8)$$

When plotted on semi-log paper these data form straight lines (figure 8).





Figs. 5 and 6. Frequency distributions of number of Washington Navel orange fruit on fruit diameter (ϕ). Yields are from the same plot of two hundred trees and are shown for three and five year intervals. Data from the Horticultural Science Department fertilizer (continuity) experiment, begun in 1917, University of California Citrus Research Center, and Agricultural Experiment Station, Riverside.

FRUIT DIAMETER AND FRUIT WEIGHT

A relationship similar to the above holds between Valencia orange volume and age. On the other hand, lemon diameters, ϕ, when plotted against fruit volumes (V) on rectilinear coordinate paper are curvilinear (figure 9). But

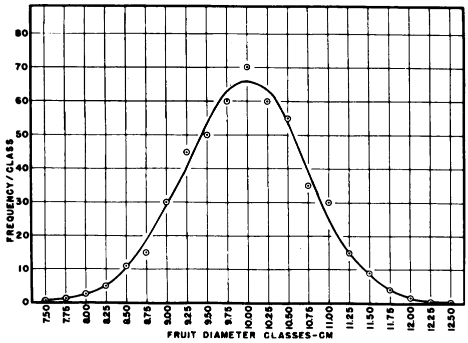


Fig. 7. Frequency distribution of number of Marsh grapefruit on fruit diameter, based on all annual reports of the Desert Grapefruit Marketing Program (Showers, 1943–50; Grunow, 1951–61).

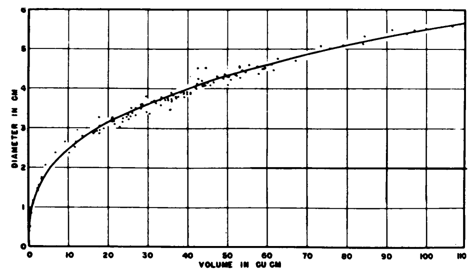


Fig. 9. Graph on arithmetic rectilinear coordinate paper of equatorial diameter vs. volume of Eureka lemon fruit. California data-composite of Turrell-Harding (Saticoy, etc.) data, 1955; Turrell-Ventura, 1957–58; and Turrell-Riverside, 1962.

the curve becomes linear on log-log paper (figure 10). Fruit fresh weight (M) also when plotted against ϕ on log-log paper, produces a straight line. The equation is of the type:

$$\log \phi = \log a + b \log V \qquad (9)$$

Fruit volume versus number of fruit per box (figure 11) plots as a power function, and individual fruit weights versus number of fruit per box are expressed by equations of similar form, plotting as a curve on rectilinear coordinates (figure 12) and as a straight line on logarithmic coordinates (figure 13).

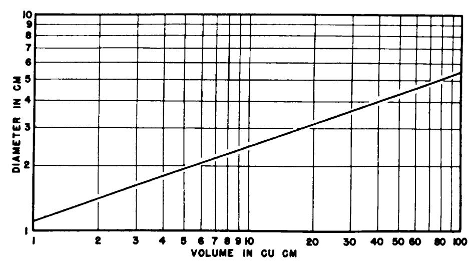


Fig. 10. Graph of the same data shown in figure 9, on logarithmic paper showing the straightening of the curve.

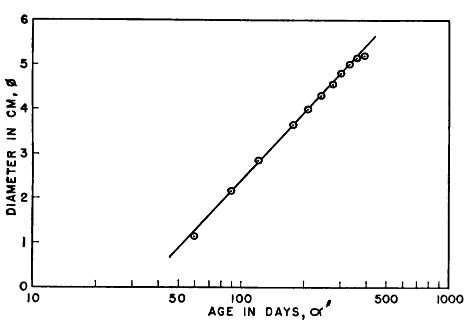


Fig. 8. Growth in diameter of lemon fruit beginning at about one-third the total age of the fruit until yellow-ripe maturity. Data from Bartholomew (1923).

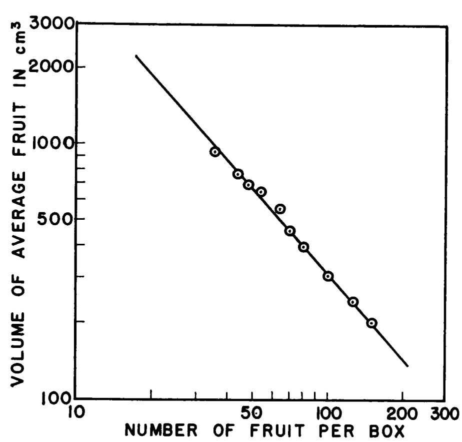


Fig. 11. Relation of number of grapefruit per box (standard, two-compartment, wooden) from Desert Grapefruit Marketing Program 1943–48 (Showers, 1943–50; Grunow, 1951–61) and published class sizes (Anonymous, 1957).

Parameters most frequently linked by fruit crop scientists are diameter and weight when weight is intended to give an estimate of fruit surface or volume. Regression equations employing diameter measurements of fruit beginning at an early stage of development are especially valuable because the high correlation with weight yields a precise estimate of growth without having to

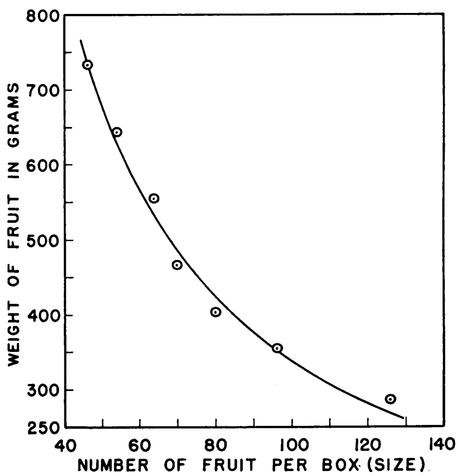


Fig. 12. Graph using arithmetic rectilinear coordinate paper showing weight of grapefruit vs. size (number of fruit per box). Data from Longfield-Smith (1935). Samples picked at different times of the year.

sacrifice the fruit. In this way, the normal growth balance in trees can be maintained. Several investigations have been made to determine constants in regression equations for apples, walnuts and oranges (Turrell, *et al.*, 1945). The relationship between the two parameters is arithmetic and linear

$$\phi = 848.31 + 124.6a \qquad (10)$$

(figure 14). It can be expressed as a power function, of course, and graphs employing log-log coordinates usually gain a small advantage from the smaller deviations in the mensuration of fruit. The constants for the first-degree equation

$$\phi = bM + a \qquad (11)$$

for four varieties of fruit are given in table 1. The second-degree equation may be readily written using a logarithmic transformation of equation 11 where $\log\,\phi = \phi'$, $\log\,M = M'$ and $\log\,a = a'$, thus:

$$\phi = bM' + a' \qquad (12)$$

the well known equation of a straight line. However, computer analysis shows equations 11 and 12 did not yield sig-

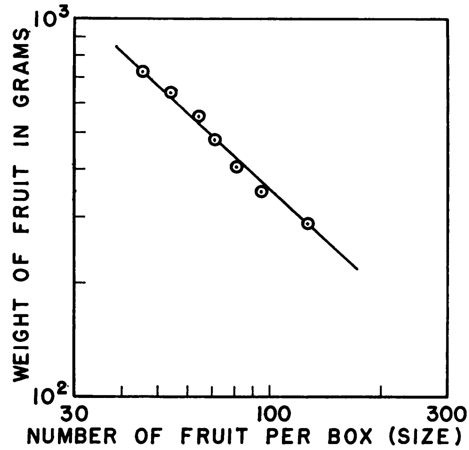


Fig. 13. Same data as shown in figure 12. Graph on logarithmic paper, showing linear curve. Weight of grapefruit vs. number per box (size). From data of Longfield-Smith (1935).

nificantly different regressions for the citrus fruit used in table 1.

The arithmetic function becomes inferior to the power function if fruit changes shape or density or both during the growth period. Neither weight nor volume increase may be constant, and as a consequence deviations from arithmetic relations of the first-degree equations may become large. For example, if a fruit of a given variety is spherical throughout its growth period we can write that the relationship between diameter ϕ and fruit volume V is constant, thus

$$4/3\pi(\phi/2)^3 = V. \qquad (13)$$

Where the volume is to be related to

TABLE 1

CONSTANTS FOR THE FIRST-DEGREE LINEAR EQUATION ($\phi = bM + a$) RELATING EQUATORIAL FRUIT DIAMETER (ϕ) IN CM AND FRUIT FRESH WEIGHT (M) IN GRAMS. BASED ON MEANS OF TEN FRUIT FROM EACH OF TEN TREES FROM EACH OF THREE CLIMATIC LOCATIONS. TURRELL-MONSELISE DATA, 1964.

Variety	a	
Eureka lemon	0.18020	0.40891
Marsh grapefruit	0.032309	0.35971
Valencia orange	0.052813	0.36721
Valencia orange*	0.10582	0.35221
Washington Navel orange†	0.10143	0.35602

* Individual fruit.
† Two climatic districts.

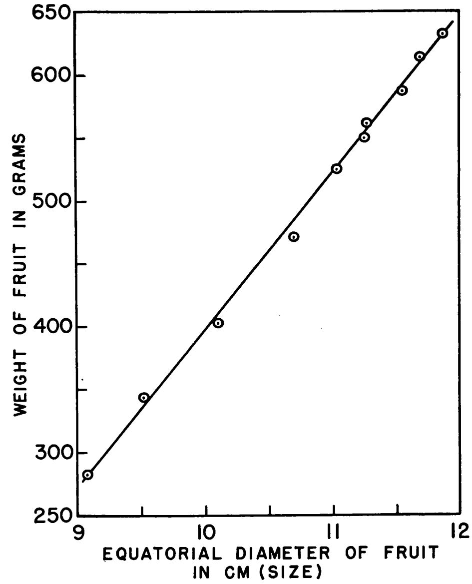


Fig. 14. The arithmetic increase in weight with diameter of Marsh grapefruit. Based on data from Harding and Fisher (1945). Fruits were picked in ten different months, at seven locations, for four years, with approximately 100 measurements of weight and 25 measurements of diameter at each sampling.

weight (M), the density factor (ρ) i.e. mass/volume must be employed and, when constant, gives

$$4/3\pi(\phi/2)^3\rho = V\rho = M. \tag{14}$$

But if $\phi/2 \neq r$, but $\phi/2 = a$, or $\phi/2 = b$, then if a is the major semi-axis and b the minor,

$$V = 4/3\pi ab^2 \tag{15}$$

and the fruit is a prolate spheroid, or if

$$V = 4/3\pi a^2 b \tag{16}$$

the fruit is an oblate spheroid. Because, with only a few exceptions, "diameter measurements" of fruit are equatorial diameter measurements, a prolate fruit such as a lemon is one in which the polar (stem-stylar) diameter $2(\phi/2) = 2a > 2b$ where $2b$ is the equatorial diameter. The volume of a spherical fruit such as an orange would be 113 when $2r = 2a = 2b = 6$ as compared with a $V = 132$ for prolate spheroid where $2a = 7$ and $2b = 6$. For an oblate spheroid such as a grapefruit where $2a = 6$ and $2b = 5$, $V = 94.2$ (Turrell tables, 1946) and where $2(\phi/2) = 2a > 2b$ where $2a$ is the equatorial diameter. Clearly it can be seen that equatorial diameter measurements may mean quite different things in terms of fruit volume or fruit weight even if the density is constant.

Fruit density, however, may vary because of density changes in peel or pulp during growth. Density differences also may be the result of the action of one or more of many factors such as varietal or strain differences, fertilizer excesses or deficiencies, pest control practices, and climatic differences. All may affect peel thickness. Other environmental factors such as freezing and structural factors such as granulation may affect pulp concentration. Turrell and Slack (1948), found a range of 20 percent in the specific gravity of citrus fruit which varied from 0.789 to 0.969, and they cite literature which also records wide ranges.

WOODY FRAME, BRANCH SIZE AND BRANCH NUMBER

Growth in fresh weight (M) of the trunk of citrus trees of various varieties, and the fresh weight of the branches, leaves, and fruit produce straight lines against tree age (α in years) when plotted on log-log paper, as shown in figures 15, 16, and 17. Also growth in dry weight (m) of leaves or woody parts when plotted against tree age (α) on logarithmic coordinate paper yield straight lines (Turrell and Austin, 1965).

A study of the defoliated woody frames of grapefruit trees in Texas which were 0.25, 0.58, 4, 7 and 32 years old showed, after they were cut up into all their branches (both straight and side), that the logarithms of branch diameter (ϕ), number of branches (n), branch surface area (S), branch volume (V), and branch dry weight (m), all increased linearly with the logarithm of tree age (α). Thus, growth in any of these dimensions can be expressed by an equation similar to number 9. The dry density (ρ) of the wood varied inversely with the age of the branch. The wood of the trunk was the least dense and that of the new mature branches the most dense. The frequency-size distribution of the branches was entirely different from that of leaves or fruit. The branches in largest number were the small growing tips, and the number of branches decreased as the diameters increased until the value 1 for n was reached for the trunk, the woody part of the tree having the greatest diameter. The total number of branches, including the trunk, on the 0.25-year-old tree was 5, while that on the 32-year-old tree was 95,431. The dry weights of the woody frames increased from 19.26 grams to 346,115 grams (Turrell, *et al.*, 1965).

According to Rashevsky (1943), the shape of a plant may be approached by the following equation

$$M = \rho\ (l_o r_o^2 + nlr^2). \qquad (17)$$

We have modified his equation using somewhat similar symbolization. Thus the dry weight m, of the woody frame of the youngest grapefruit tree we have measured which is shown in figure 18, is

$$m = \rho\ (l_7 r_7^2 + n_o l_o r^2). \qquad (18)$$

In this equation the first term refers to the trunk as does the first term in the Rashevsky equation (equation 17) and in both the second terms refer to the branches. In equation 18 where l_7 is the length of the trunk and r_7 is the radius of the trunk, l_o is the average length of the branches and r_o is the average radius of the branches of our 0.25-year-old tree (figure 18). For the trees of four to 32 years of age (figure 19), six terms must be inserted in equation 18 between $l_7 r_7^2$ and $n_o l_o r_o^2$ to account for the growth of the six classes of branches of different diameters. The average dry density of the wood of the entire tree, ρ, must be changed to a specifiic average density for each class-size of branch and each term must include its own specific density factor. Similar terms have been added to account for fruit.

It is possible at the present time to describe a growing citrus tree or any of its above-ground parts in terms of weight, size, or number without destroying the tree or any of its parts. Deviations can be expected in the calculations as compared with what might actually be found. Trees grow at different rates in Arizona, California, Florida, and Texas but the laws of growth are now clear, and it is necessary only to find the relationship of two pairs of parameters for the trees of the four states to be compared. The log-log linear relationship for a given location may be translated into a more precise equivalent relation-

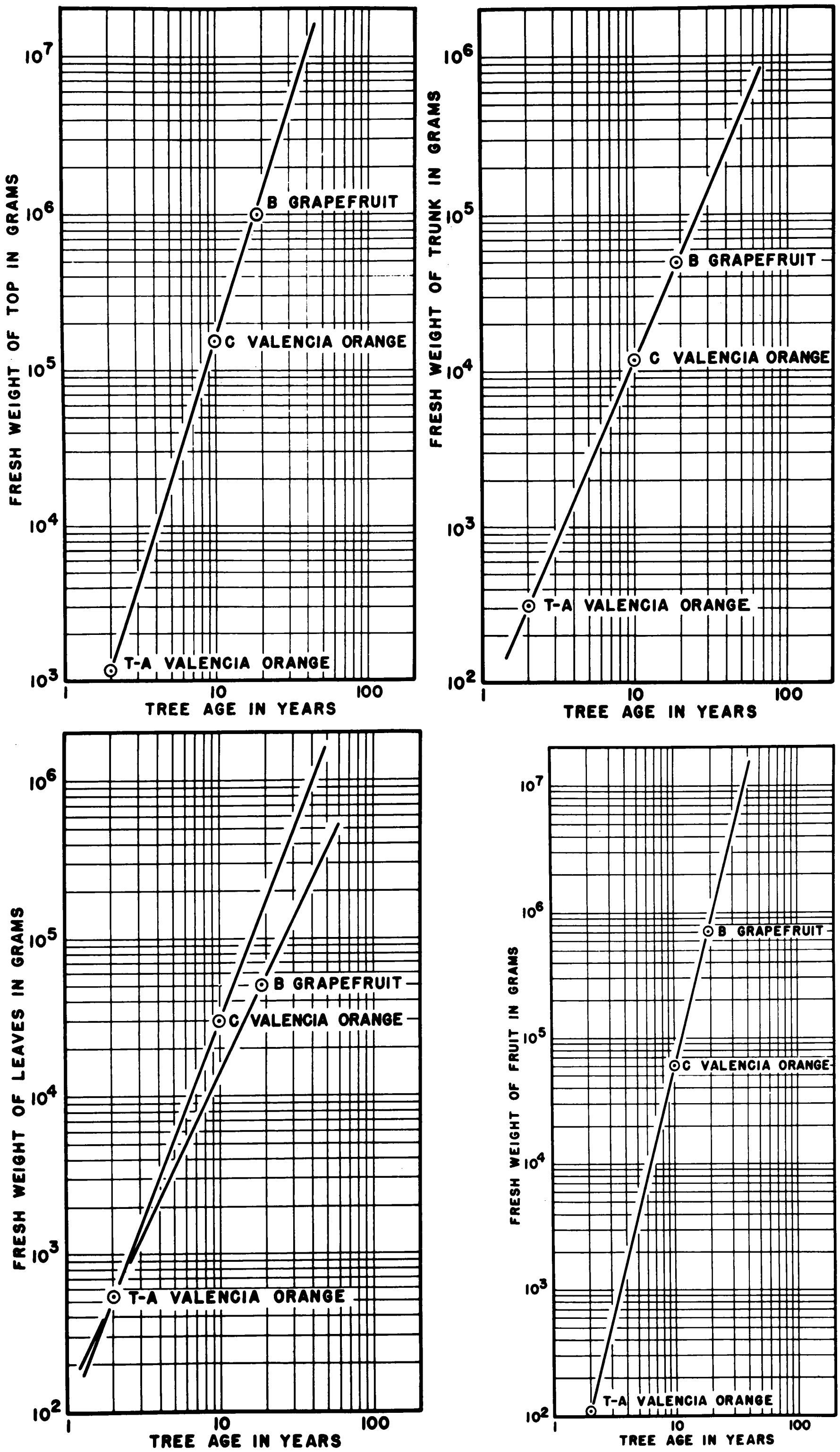


Fig. 15. Legend on bottom of page 439.

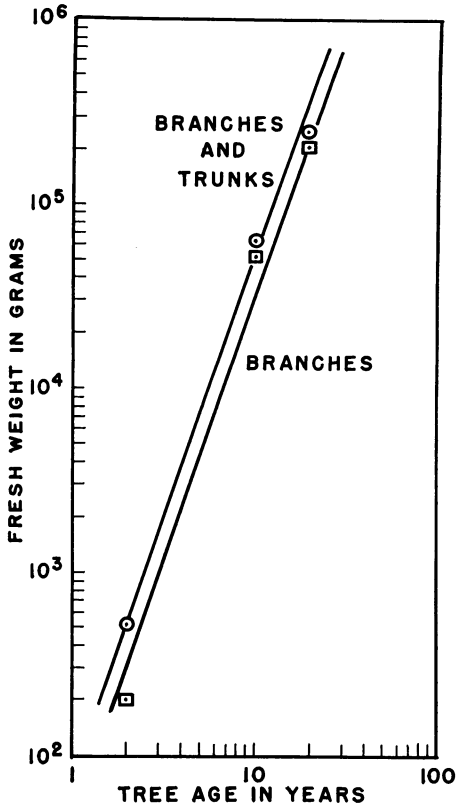


Fig. 16. Growth in fresh weight of above-ground woody parts (branches, branches and trunks) of a two-year-old Valencia orange, ten-year-old Valencia orange, and a 19-year-old grapefruit tree. Data from Turrell and Austin, unpublished; Cameron and Appleman (1934) and Barnette, *et al.* (1931).

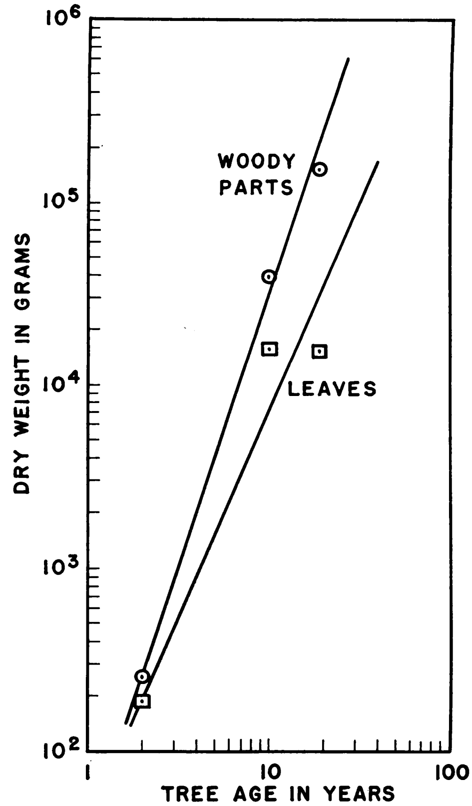


Fig. 17. Growth in dry weight of above-ground woody parts (trunks and branches), and leaves a two-year-old Valencia orange, ten-year-old Valencia orange, and a 19-year-old grapefruit tree. Data from Turrell and Austin, unpublished; Cameron and Appleman (1934); and Barnette, *et al.* (1931).

ship by correcting for the relative growth rates in the two locations. The correction ratio applicable can be derived from table 2 adapted from Cooper, *et al.* (1963).[2]

A rather crude but much more general example is provided if the tree height and trunk diameter data gathered in various parts of the world by Webber and Batchelor (1943) is graphed using log-log coordinates. Surprisingly good fits to a straight line are shown despite the numerous permutations and combinations of variety, soil, climate, and cultural practices, world wide (figure 1), of which a minimum of 3.67×10^{10} affect the growth of the tree (Turrell, *et al.*, 1964). Thus, growth studies already in the literature provide a basis for easy, quick, and more or less precise comparisons of growth as affected by many factors.

[2] More precise correction ratios could be obtained from the log mean of two or more year's data. This is presently unavailable but will be published at a later date.

Fig. 15. Growth curves of citrus trees and citrus tree organs based on composite data from a two-year-old Valencia orange tree, T-A, Turrell and Austin, University of California Citrus Research Center Project No. 1731, 1962, unpublished; an average of three ten-year-old Valencia orange trees, C, Cameron and Appleman (1934); and one nineteen-year-old grapefruit tree, B, Barnette, *et al.* (1931).

TABLE 2

SEASONAL INCREASE IN CROSS-SECTIONAL AREA OF TRUNKS OF VALENCIA ORANGE TREES GROWING AT SEVEN LOCATIONS DURING 1961

	Orlando	Claremont	Weslaco	Tempe	Indio	Riverside	Santa Paula
				cm²			
Early spring (Feb., Mar.)	0	1.28	0	0	0.60	0	0.29
Late spring (Apr., May)	5.10	3.55	2.10	3.80	6.70	0.35	0.31
Early summer (June, July)	10.57	6.32	0.90	7.20	6.00	4.70	4.80
Late summer (Aug., Sept.)	14.49	6.31	1.20	7.00	6.40	3.65	9.40
Fall (Oct., Nov.)	1.88	3.05	0.01	3.10	5.40	4.13	2.67
Winter (Dec., Jan.)	0.99	0	0.90	0.30	−0.90	0.48	0
Total	33.03	20.51	5.11	21.40	24.20	13.31	17.47

ROOT DENSITY AND YIELD

To the authors' best knowledge, no curves or equations are now known for root growth which relate tree-age to wet weight, dry weight, surface area, number of roots per size class, or other parameters important to the physiology of the tree. It seems probable that a linear relation exists between log of tree age and the log of root parameters such as density in the soil, weight, surface, and volume because our graphs and computer tests of equations of best fit of the work of Cahoon, *et al.* (1959) show that there is a linear relation between log of yield ($\hat{Y}$) and log of density of roots per unit volume of soil (X), as shown in figure 20.

$$\hat{Y} = 10.54 \log X - 9.16 \log X^2. \qquad (19)$$

However, the equation of best fit was

$$\hat{Y} = 968.60 \log X. \qquad (20)$$

And as shown in figure 15, there is a linear relationship between log tree age and log yield. It seems likely that plots of log of stem diameter or height vs. log of a given root parameter will produce efficient methods of obtaining requisite sizes of root systems because the log of stem weight vs. log of root weight is a linear relationship (Huxley, 1932; Pearsall, 1927; re: cotton, peas, carrot, turnip, and others).

Frequency distribution data is not available for citrus roots despite the relatively large number of root studies that have been made. However, it may be conjectured that an inverse power function will express a frequency-diameter distribution in roots just as it does in branches. Lacking citrus-root data for analysis, a somewhat similar (but not inverse) function is shown for the elm tree (*Ulmus pumila*). The log of the number of roots per class plots a straight line against the log of the total length of roots per class (figure 21).

DISCUSSION

Elsasser's (1964) organismic theory unqualifiedly assumes the validity of the laws of ordinary quantum mechanics for the physical and chemical processes

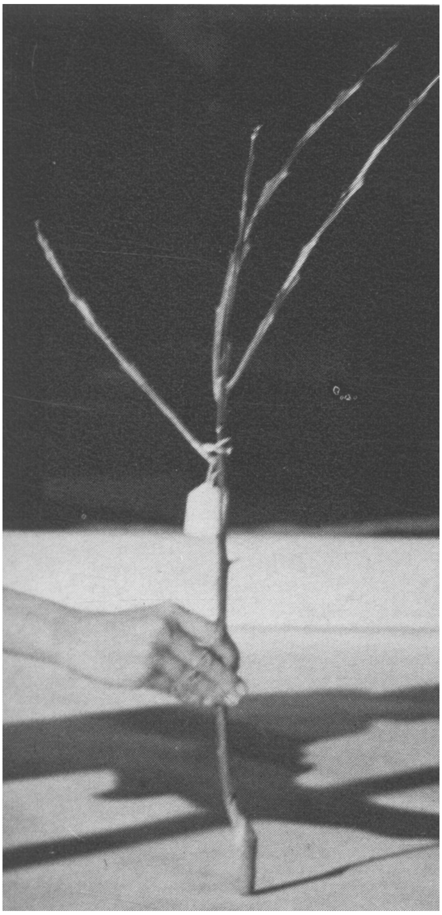


Fig. 18. One 3-month-old (0.25 year) grapefruit tree of four trees from budding, which were averaged for growth parameters. Note the long scaffold branches which will become the oldest branches with the longest internodes. From Turrell, *et al.* (1965).

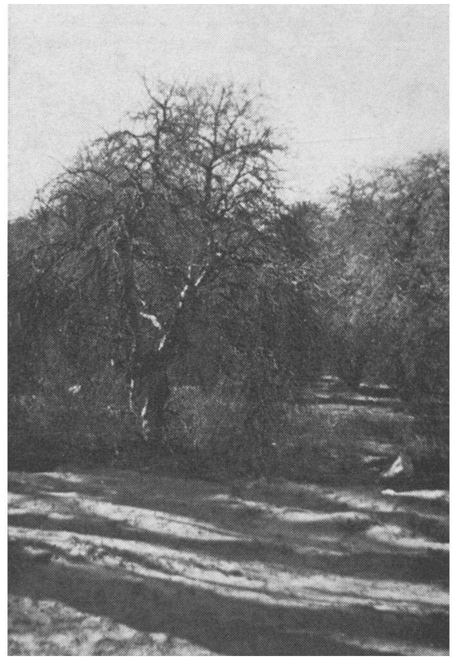


Fig. 19. Old grapefruit trees showing the large amount of very fine, small, short terminal growth.

going on in the organism. The growth of the tree as a whole or the growth of its parts reflects the physical processes involved. Many of the physical processes, we know, are based on kinetic theory, the hypothesis that all molecules are in motion. It is noteworthy that relatively large numbers of pairs of parameters for growing citrus trees are exponential or power functions and give linear plots on semi-log or log-log graph paper, as shown herein. Many not mentioned do likewise. That a plant is a heat engine has long been recognized. Photosynthesis, respiration, and growth require specific temperature ranges usually narrower than the interval between 0° and 55°C. Thus we find that the penetration of cells of the root by water (Mazur, 1965), evaporation (Boelter, *et al.*, 1946) or transpiration in still air (Turrell and Austin, 1966; Turrell, 1965), diffusion of water vapor from stomata of different degrees of opening (Ting and Loomis, 1965), absorption of light by chlorophyll leaf pigments (Turrell, 1939; Turrell and Waldbauer, 1935; Benedict and Swidler, 1961) and by leaf carotenes (Zscheile and Porter, 1947), thermal conductivity of citrus wood and fruit (Turrell, *et al.*, 1967), the rates of chemical reactions (Johnson, *et al.*, 1954), quantity of adsorption with pressure or concentration (Millard, 1937; Turrell, *et al.*, 1955), and incorporation of P^{32} into the nucleus (Pauling and Hanawalt, 1965; Turrell, *et al.*, 1955),

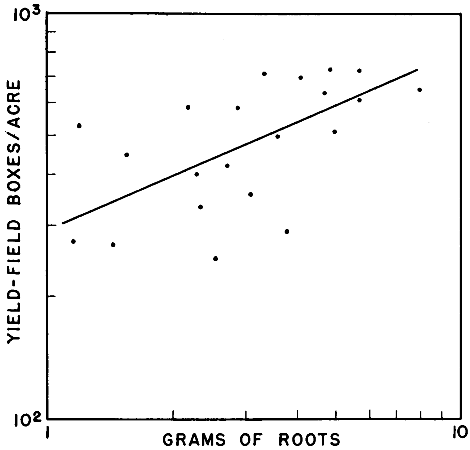


Fig. 20. The log of yield of Washington Navel oranges (field boxes/acre) from the Citrus Experiment Station continuity plots, gives a straight line against the log of the grams of fresh roots per 450 n³ of soil in a depth of 0 to 3 feet, Cahoon, *et al.* (1959).

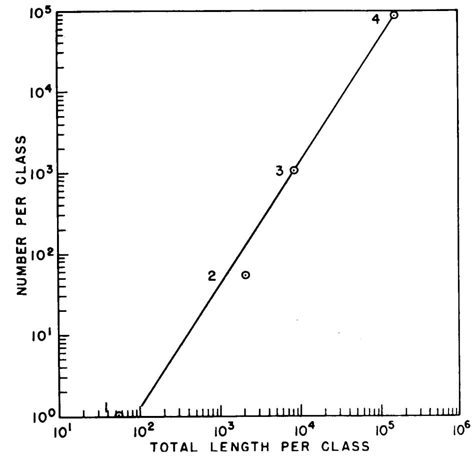


Fig. 21. The logarithm of the number of elm roots (*Ulmus pumila*) per class plotted against the logarithm of the total length (cm) of roots per class (combined). Class 1 are main roots arising from the base of the plant; Class 2 are secondary roots arising from the main roots; Class 3 are tertiary roots arising from secondary roots; Class 4 are quaternary roots arising from tertiary roots. Data from table 118, Handbook of Biological Data (1956) by permission W. B. Saunders Co.

are described by power or exponential equations. These and many other physical and chemical processes seem to be reflected in linear log-log relations of root size (weight or density) and age, stem diameter, stem height, tree surface, and yield versus tree age, leaf area and fruit diameter, and age of the plant part. These relationships appear to be a result of the kinetic characteristics of atoms and molecules such as mass, velocity, dimensions, vibration-frequency, rotation, and others. Insofar as rates are concerned, the quantitative mathematical relations are similar to those of the collision theory developed by Arrhenius (1899) for multiple hit processes.

The nature of the curves and the equations presented here describe growth as a process of self-multiplication, which speeds up with increasing age or its equivalent, size, and also responds to an external environment with built-in specifications of the kinetic and collision theories of the behavior of matter wherein the principal force is temperature. The building blocks, of course, are the molecules and atoms of the aerial and subterranean environment. The development rate of a citrus tree growing equally in all its organs is at any instant proportional to the size of the tree, y. The rate of change of y is proportional to y itself. Then y is an exponential function of x, and a and b are constants. Thus, the general equation is:

$$y = ae^{bx}$$

In the orchard, citrus trees show aging to be a logarithmic process. It has been assumed that the linear nature of the log of tree-parameter plotted against tree age results from the logarithmic growth of roots and the logarithmic growth of the entire top of the tree which has resulted from a logarithmic decrease in the supply of water and of sunlight (Turrell, 1961). Lone trees should grow semi-logarithmically with time, in a way similar to cucumbers grown under adequate light in the greenhouse (Gregory, 1921). Freeze injury to trees under marginal conditions should be found to be a logarithmic process (power function) inasmuch as Camp (1965) has shown that the log of growth velocity of ice on several surfaces (glass, lucite, aluminum) is a function of the log of the differences between freezing and under-cooling temperatures and Salt (1958) showed that the relationship between the log of the probability and time required to freeze a given number of insects is a linear function of the log of the numbers in the population. Some temperature-dependent insecticides, such as elemental sulfur, volatilize exponentially with temperature (Turrell, 1947), while liquid and gaseous sulfur compounds penetrate plant structures, and combine with proteins logarithmically with osmotic pressure or vapor pressure, and gas pressure (Turrell, *et al.*, 1955) and thus the amount of damage is a logarithmic function of temperature.

SUMMARY

Citrus trees and citrus tree parts follow curves of growth similar to those of nonwoody plants. The central axis of the "drawn-out" $\int$-curve of growth produces a straight line on logarithmic graph paper (log-log) and can be readily used to predict the amount of growth within any interval. The number of individual tree parts, when plotted against the size of the parts, gives a "frequency-distribution" curve which is *normal* (bell-shaped) for leaves, may be *normal* for fruit only if very large numbers are involved, and is an *inverse power function* for branches. A relatively large number of physical or physico-chemico processes underlying tree growth are linear semi-log (logarithmic or exponential) or log-log (power) functions.

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