PREDATOR / PREY INTERACTION MODELS

Notes

I. Lotka-Voltera Predator-Prey Model

A. This simple model is based on 2 simple propositions:

1. The birth rate (B_1) of the predator (N_1) will increase as the number of prey increase; and
2. The death rate (D_2) of the prey (N_2) will increase as the number of predators increase.

The model is composed of 2 equations:

$$dN_1/dt = (B_1 N_2 - D_1)\, N_1 \qquad (1)$$

which calculates predator population growth where B_1 = the birth rate of the predator and D_1 = the death rate of the predator. The individual birth rate of the prey is not directly dependent on the abundance of the predator, but its death rate is dependent upon the predator's density. Thus, the birth rate (B_2) of the prey is a constant. The growth rate of the prey population is described by the following equation:

$$dN_2/dt = (B_2 - D_2 N_1)\, N_2 \qquad (2)$$

where D_2 = the death rate of the prey.

B. Note that:

1. The predator birth rate depends upon the food (prey) supply available; and
2. The death rate (D_1) of the predator is not dependent on the prey density.

C. See information box on next page.

D. Predation is only one of several agents that cause population cycles. Other factors implicated include mass emigration; physiological stress due to overcrowding; and genetic changes in the population.

E. Population cycles are difficult to achieve in the field or laboratory. Usually the predators search out every one of the prey and then they go to extinction due to lack of food. Lotka-Volterra equations are too simple for practical use.

F. The Lotka-Volterra equations can be improved by some minor, but realistic changes in the "zero-growth" curve of the prey. See information box on the next page.

G. Other improvements can be made by inclusion of a "refugium" in the system. A refugium is a place where the prey may take "refuge" from the predator. Refugium occur commonly in nature and probably account for a great many of the balanced predator-prey systems that exist. A refugium may not necessarily always be a simple area that the prey finds and colonizes before the predator is able to follow it.

LOTKA-VOLTERRA PREDATOR / PREY MODEL

The predator/prey models (equations) of Lotka and Volterra are based upon two very simple propositions: the birth rate of the predator (N_1) will increase as the number of prey (N_2) increases, while the death rate of the prey will increase as the number of predators increases. The individual birth rate of the predator depends upon the food that is available, which in turn depends upon the density of the prey population. The death rate of the predator is not dependent on the prey density. The following equation is for the total growth of the predator population:

$$dN_1/dt = (B_1N_2 - D_1)\ N_1$$

$$= B_1N_1N_2 - D_1N_1$$

where B_1 = the birth rate of the predator and D_1= the death rate of the predator. The individual birth rate of the prey is not directly dependent on the abundance of the predator, but its death rate is dependent upon the predator's density. Thus, the birth rate (B_1)the prey is a constant. The growth rate of the prey population is described by the following equation:

$$dN_2/dt = (B_2 - D_2N_1)\ N_2$$

$$= B_2N_2 - D_2N_1N_2$$

where D_2 = the death rate of the prey.

When the two populations are in equilibrium, the predator density (N_1) equals the prey's birth rate divided by the prey's death rate (= B_2/D_2), and the prey's density (N_2) equals the predator's death rate divided by the predator's birth rate (= D_1/B_1). This this is shown graphically below.

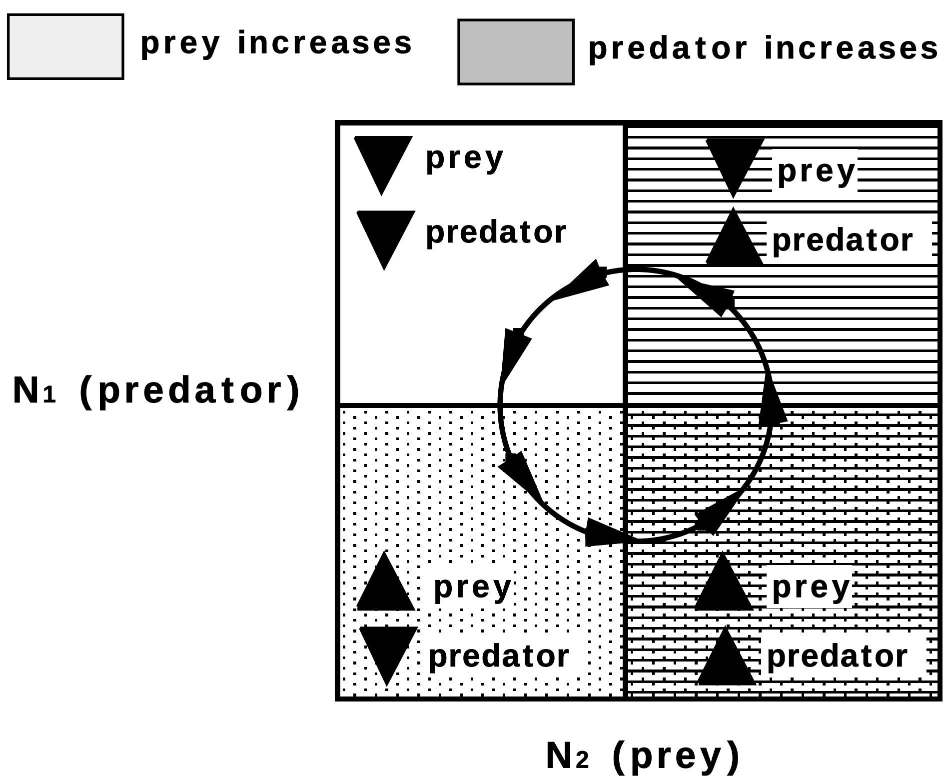

Fig. 11.1. **PREDATOR - PREY INTERACTION** as depicted by the Lotka-Volterra equations. The graph above shows the joint abundance of the two interacting populations. Modified from Wilson and Bossert (1971).

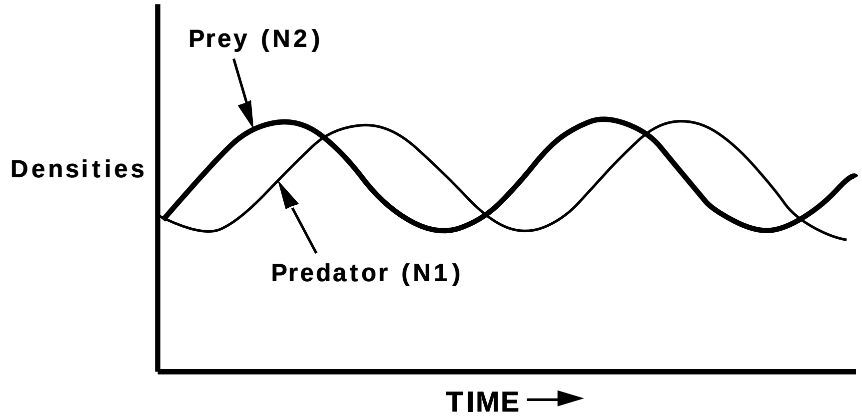

Fig. 11.2. **PREDATOR - PREY INTERACTION** as depicted by the Lotka-Volterra equations. The graph above shows the result when the abundances of the two species are plotted as a function of time. Modified from Wilson and Bossert (1971).

LOTKA-VOLTERRA PREDATOR/PREY MODEL
- Stabilization -

The predator-prey model may be stabilized by making two assumptions about the growth rates of the prey and one assumption about the growth rate of the predator. These assumptions are graphically illustrated on the following graphs. Notice that the horizonal line associated with zero prey growth has been turned downwards at both ends so that a "hump" is formed. Mathematically this hump indicates that the growth rate of the prey declines when the prey population is very low (because of inability to find mates) and when very high (overcrowding of prey). Thus the prey population growth rate decreases at low densities and at high densities. Notice also that the vertical line which represents predator growth is bent to the right side at the top. This puts a limit on the growth of the predator population when prey are at very high densities. As long as the vertical predator line stays to the right side of the top of the "hump" (see 2 graphs below), the model will be stable (oscillations remain same size or become smaller).

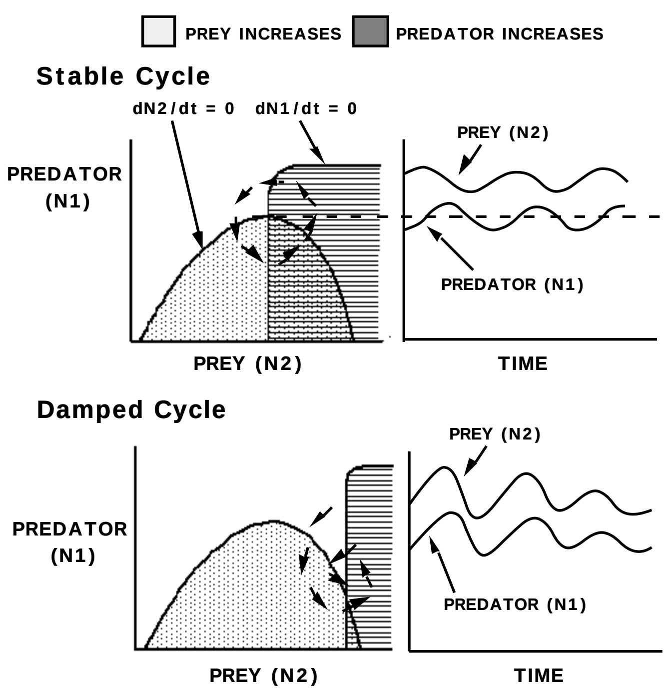

Fig. 11.3. **Growth curves of predator and prey populations.** According to the theory, major changes in the predator-prey oscillationsare are obtained when the relative positions of the zero-growth curves are changed. The arrows indicate the directions followed by the joint abundances of the interacting populations. The graphs on the left show the relationship of the predator and the prey in which each increases and decreases in long-range equilibrium. On the right, the relationships are shown as they change through time, producing the familiar population cycles. Modified from Wilson and Bossert (1971).

LOTKA-VOLTERRA PREDATOR/PREY MODEL
- Incorporation of a Refugium -

Notice in the upper right-hand graph below that the vertical line is to the right of the hump and the model is unstable (continually greater oscillations). This can be stabilized by creating a "refugium" indicated on the lower right-hand graph as the modification to the prey zero growth line along the side of the Y axis (Predator Density). As long as the prey can be protected from the predator when the prey densities are low, then the prey will not be totally eliminated. This happens in nature in two ways: 1) prey are protected in areas that predators overlook or cannot obtain access to them; and 2) prey escape high concentrations of predators by emigration to areas of lower predator density.

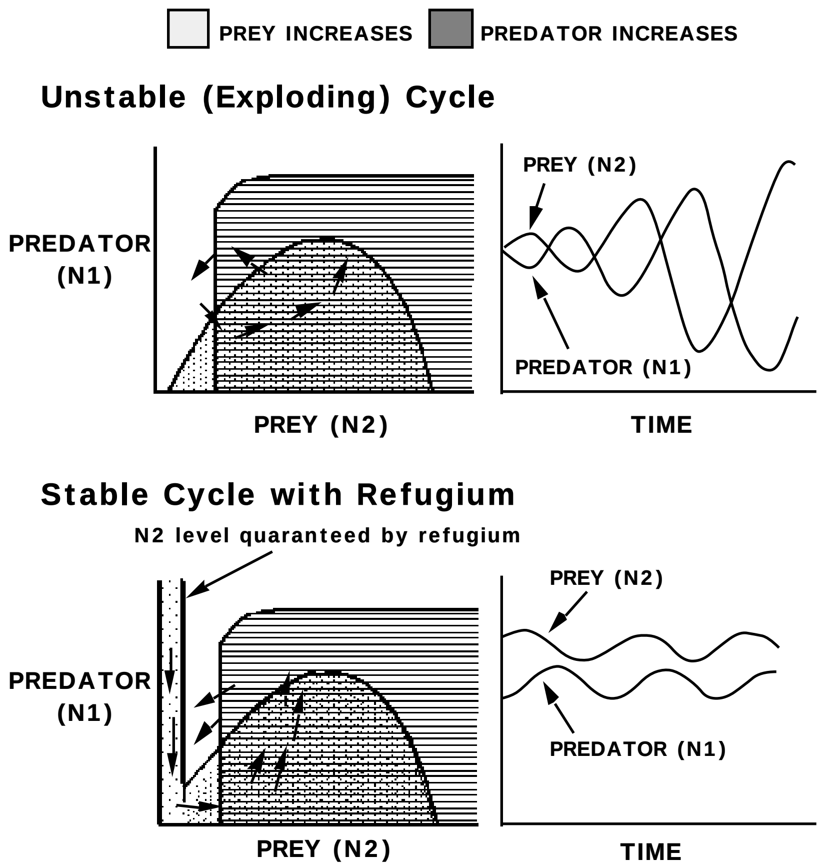

Fig. 11.4. **Two additional kinds of oscillations in predator-prey systems.** Modified from Wilson and Bossert (1971).

Notes

H. Volterra's Principle: If two species (predator/prey, parasitoid/host) are destroyed at the same rate by some outside agency (such as indiscriminate hunting or the use of broad spectrum pesticides by man), the prey will proportionately increase and the predators will proportionately decrease. This is because:

1. The birthrate of the prey is not affected;
2. The deathrate of the prey is reduced; and
3. The birthrate of the predator is reduced.

QUESTIONS

1. What propositions are the Lotka-Voltera predator-prey models based upon?
2. What is a major fault of the original Lotka-Voltera predator-prey models? How can they be improved?
3. The zero-growth lines of the prey and predator may be modified to stabilize the model, does this have any relationship to the real world?
4. What is Volterra's Priciple? How does this principle play a role in the suppression of insect and mite populations in agroecosystems?

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Chapter 18: pp. 367–398, **Van Driesche, R. G. and T. S. Bellows, Jr. 1996.** Biological control. Chapman and Hall, New York. 539 pp.