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Two-phase flow of water and air during aerated subsurface drip irrigation

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Abstract

Here, we investigate the two-phase flow problem as in aerated subsurface irrigation. We extend McWhorter's one-dimensional equation for the concurrent flow of air and water (CEFAW) to three dimensions, and present explicit solutions subject to linear functions for the two-phase unsaturated hydraulic conductivity, diffusivity and fractional flow function. We present both steady- and unsteady-state solutions to the CEFAW corresponding to two types of constant continuous sources (a line source and a point source), which are relevant to aerated subsurface irrigation using emitters. The two-dimensional solution appears as the modified Bessel function of the second kind of order zero while the three-dimensional one as a complex exponential function. Graphic illustrations of the mathematical solutions indicate asymmetrical distributions of water content around the supply source due to gravity at the steady state established at large time, which differ in patterns and magnitudes for the two types of supply sources.

Keywords: Two-phase flow; Airflow; Water flow; Subsurface irrigation; Mathematical models

1. Introduction

In this paper, we investigate aerated subsurface irrigation, i.e. the concurrent flow of water and air. We nominally name this method 'oxygation' as compared to 'fertigation' or 'chemigation' when fertilizers are added to the irrigation water supply. Oxygation is a typical two-phase flow problem.

Irrigation is an age-old practice and has been continuously refined to meet different requirements. While beneficial to plants, irrigation, when performed improperly, may have unwanted side effects on the environment such as irrigation-induced salinity in soils, groundwater and surface water bodies. In addition to those side effects, water applied during irrigation may not necessarily assist plants to function properly and achieve increased yield as expected.

During an irrigation event, with whatever delivery technique, the purging of the air in the soil by the infiltrating front of water will create a verifiable level of anaerobic condition in the irrigated zones (Sojka, 1992; Drew and Stolzy, 1996; Arteca, 1997; Rawler et al., 2002; Blokhina et al., 2003). Consequently, by reducing oxygen concentration in the soil as it is

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irrigated, root metabolism is reduced in the zones where the irrigation waterfront arrives. As a consequence, this type of irrigation plays a passive role not only as the water supplier, but also constraining the functioning of crops and microbial communities. The intended benefits of irrigation, therefore, can be undermined by the imposed oxygen deficit in the soil through the creation of an unfavourable anaerobic condition. Even though this anaerobic condition is temporary, plant and microbial metabolism may be constrained. Such a decline in metabolism due to oxygen deficit is possibly responsible for an inefficient uptake of water during irrigation, resulting in added drainage and water loses through other routes such as evaporation. There is a large body of published work on the experimental evidence of the decline in metabolism due to a lack of oxygen which relates to the effect of permanent anaerobic (flooding) conditions on soil microbiology and root functions (Veen, 1988). However, little work exists on the effects of temporary flooding experienced during irrigation.

While the reported studies on irrigation in the literature so far fail to offer an option for substantial reduction in water use while maintaining crop production, in a recent report on glasshouse and field experiments, Bhattarai et al. (2004) confirmed that dramatic increases in crop yields, water use efficiency and salinity tolerance can be achieved with the use of oxygenated subsurface drip irrigation water, especially for crops grown on heavy clay soils. The research by Bhattarai et al. (2004) showed that for soybean, oxygation increased water use efficiency (WUE) by 54 and 70%, respectively, for hydrogen peroxide application and air injection using a venturi valve, and pod yield by 82 and 96%, respectively, for the two treatments. Likewise, for crops grown across a range of saline soil conditions, aeration using the venturi principle resulted in yields superior to those of the non-aerated controls (Pendergast and Midmore, unpublished, 2003). Benefits of aeration using the venturi principle in California (Goorahoo et al., 2002), or using hydrogen peroxide in Germany (Heuberger et al., 2001) on crop growth are also reported.

Aeration of subsurface drip irrigation water, using appropriate techniques such as the venturi principle, hydrogen peroxide, or even a twin vortex system, could be potentially the most significant recent approach to economise on large-scale water usage and minimise drainage in irrigated agriculture.

Oxygation is a two-phase flow problem. The classic issue of two-phase flow is an important topic in other fields such as chemical and petroleum engineering. In hydrology and soil physics, the issue of two-phase flow was raised by Green and Ampt (1911), and then after pursued only by a few researchers (Powers, 1934; Baver, 1937; Lewis and Powers, 1939; Elrick, 1961). But the effect of airflow on water movement in irrigated and non-irrigated soils has been largely neglected in subsequent studies over the past six decades. The revigoration following six decades of neglect of the quantitative studies of water movement and associated soil air effects was mainly due to Morel-Seytoux and his co-workers. The representative work include Brustkern and Morel-Seytoux (1970), McWhorter (1971), Morel-Seytoux (1973, 1983), Vachaud et al. (1973, 1974), Parlange and Hill (1979), Parlange et al. (1982), Sander and Parlange (1984), Sander et al. (1984, 1988a,b,c), and Philip and van Duijn (1999). Those studies were either theoretical or undertaken in the laboratory. While using the Burgers equation and high resolution recorded data using time domain reflectometry (TDR) to investigate flow patterns in a field soil, Su et al. (2004, p. 5) found that the effect of airflow on soil water movement in the topsoil is obvious due to large air-filled porosity. All these studies, however, were limited to the analysis of soil water movement as affected by air in non-irrigated soils, without considering important issues such as water saturation and air purge, anaerobiosis and subsequent gradual recovery of natural soil water–air relationships under irrigation conditions, and, surprisingly, the presence of plants.

The aim of this paper is to gain further understanding of the phenomenon of oxygation at the hydrological and soil physical levels by the quantitative analysis of concurrent air and water flow during subsurface drip irrigation.

2. Mathematical formulation

The equation governing the one-dimensional flow of water affected by airflow is given by McWhorter (1971, p. 53, Eq. (44)). Sander et al. (1988c) rewrote

McWhorter's formulation in the following form with notation more familiar in hydrology and soil physics literature

$$\frac{\partial\theta}{\partial t}+\frac{\partial q}{\partial z}=0 \tag{1}$$

where

$$q=-D(\theta)\frac{\partial\theta}{\partial z}+CK(\theta)+V(t)f_{\mathrm{w}}(\theta) \tag{2}$$

$$V(t)=q_{\mathrm{w}}+q_{\mathrm{a}} \tag{3}$$

$$C=1-\frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}} \tag{4}$$

where

q_{w} and q_{a} are the fluxes of water and air, respectively, and ρ_{w} and ρ_{a} are the densities of water and air, respectively.

The two-phase diffusivity, $D(\theta)$, and two-phase unsaturated hydraulic conductivity, $K(\theta)$, are related to the one-phase diffusivity, D_{w} (water only), and unsaturated hydraulic conductivity, K_{w}, by the following relationships,

$$D(\theta)=D_{\mathrm{w}}(1-f_{\mathrm{w}}) \tag{5}$$

$$K(\theta)=K_{\mathrm{w}}(1-f_{\mathrm{w}}) \tag{6}$$

where f_{w} is the fractional flow function.

For concurrent water and air flow, $C\cong 1$ (Sander et al., 1988c). Eqs. (1) and (2) can be written in a usual form,

$$\frac{\partial\theta}{\partial t}=\frac{\partial\theta}{\partial z}\left(D(\theta)\frac{\partial\theta}{\partial z}\right)-\frac{\partial}{\partial z}[K(\theta)+V(t)f_{\mathrm{w}}(\theta)] \tag{7}$$

McWhorter (1971), Sander and Parlange (1984), Sander et al. (1984), and Sander et al. (1988a,b,c) extensively investigated the above one-dimensional formulation for concurrent flow of water and air.

Here, we particularly mention that Sander et al. (1988c, p. 226) give

$$\frac{\mathrm{d}i}{\mathrm{d}t}=V(t)-CK_{\mathrm{wi}} \tag{8}$$

where K_{wi} is the two-phase unsaturated hydraulic conductivity at the initial water content, θ_i, and C is given by Eq. (4).

We further refer to Rogers et al. (1983) and Sander et al. (1988c, p. 228) for

$$i=Qt \tag{9}$$

where Q is equal to the infiltration rate.

It is obvious from Eqs. (8) and (9) that we can treat $V(t)$ either as a function of time or a constant, depending on the value of Q.

The above formulation was developed for one-dimensional flow only. By extending the diffusion term in Eq. (7) into three dimensions, it can be generalised to higher dimensions,

$$\frac{\partial\theta}{\partial t}=\nabla\cdot[(\mathrm{D}\nabla\theta)]-\frac{\partial}{\partial z}[\mathrm{K}(\theta)+\mathrm{V(t)f_w}(\theta)] \tag{10}$$

where ∇ is the Laplace operator.

In this paper, we investigate the equation of concurrent flow in three dimensions, namely Eq. (10). In our analysis, we are more concerned with the effect of airflow on three-dimensional flow in a subsurface aerated drip irrigation environment (i.e. oxygation) which can be regarded as a point source.

To illustrate our analysis for oxygation where the convective flow is dominating over diffusion, we simplify the formulation appearing in Eq. (10) by establishing the following conditions:

1. constant diffusivity, D, which is a practical approximation due to strong convection under field conditions;
2. unsaturated hydraulic conductivity, $K(\theta)$, given by
$$K(\theta)=K_1+K_2\theta \tag{11}$$
3. the fractional flow function given by
$$f_{\mathrm{w}}(\theta)=f_1+f_2\theta \tag{12}$$
4. with a constant drip irrigation rate, V to represent $V(t)$.

Under these conditions, Eq. (10) becomes

$$\frac{\partial\theta}{\partial t}=D\nabla^2\theta-\mathrm{U}\frac{\partial\theta}{\partial z} \tag{13}$$

with

$$U = K_2 + Vf_2 \quad (14)$$

Equation (13) is a linearised advection–dispersion equation, or Fokker–Planck equation and its solutions with different initial and boundary conditions are readily available. Here, it should be emphasised that Eq. (13) governs the two-phase flow in an unsaturated soil, despite its similarity to the conventional Fokker–Planck equation encountered in soil physics (Philip, 1969) and to other environmental models (Su, 2004).

In the following sections, we present solutions of Eq. (13) with examples to illustrate their application to oxygation.

3. Solutions of the two-phase flow equation in two and three dimensions for continuous subsurface emitter irrigation

The emitter used in subsurface irrigation can be regarded as a point source. In this case, we are only interested in a semi-indefinite medium at a supply rate of Q. Under these conditions, the problem we formulated is subject to the following initial and boundary conditions.

$$t = 0 \quad r > r_0 \quad \theta = 0 \quad (15)$$

$$t > 0 \quad r = r_0 \quad V_p = K(\theta) - D\nabla\theta(r_0, t) \quad (16)$$

$$t > 0 \quad r \to \infty \quad D\nabla\theta(r_\infty, t) = 0 \quad (17)$$

where

V_p is the inflow rate;

r_0 is the hypothetical radius of the spheric source

$$r = (x^2 + y^2 + z^2)^{1/2} \quad (18)$$

The condition defined for the three-dimensional flow is similar to the one for one-dimensional flow (Sander et al., 1988a,b,c). The problem described in this case is similar to the moving point source problem given by Carslaw and Jaeger (1959, p. 266–267) except that U, the velocity in the present case, is in the z direction. Two cases are considered in this analysis.

3.1. Three-dimensional point source: point emitters

For a three-dimensional flow when the emitter can be regarded as a point source, the solution of Eq. (13) is then given by

$$\theta = \frac{Q_3}{2rD\pi^{3/2}} \exp\left[\frac{(K_2 + Vf_2)z}{2D}\right] \times \int_{r/2\sqrt{Dt}}^{\infty} \exp\left\{-\xi^2 - \left[\frac{(K_2 + Vf_2)r}{4D\xi}\right]^2\right\} d\xi \quad (19)$$

which is equivalent to Eq. (2) of Carslaw and Jaeger (1959, p. 26), with r given by Eq. (18) and Q_3 is the outflow rate given by

$$Q_3 = 4\pi r_0^2 V_p \quad (20)$$

where

V_p is the velocity of the inflow from the delivery pipe.

Equation (19) describes soil water movement subject to constant supply at a rate of Q_3 for a finite time t. Carslaw and Jaeger (1959, p. 267) showed that as $t \to \infty$, a steady water regime, θ_s, is established, i.e. Eq. (19) becomes

$$\theta_s = \frac{Q_3}{4\pi Dr} \exp\left[-\frac{(K_2 + Vf_2)(r - z)}{2D}\right] \quad (21)$$

The steady-state solution in this situation describes the steady moisture profiles developed for a large time. A steady-state profile implies $d\theta/dt = 0$ at a particular point while the water movement continues.

3.2. Two-dimensional line source: supply line emitters

When water is supplied at a flow rate of Q_2 per unit length as a line source along the y-axis, the soil water content established at a steady state at the point (x, y, z) can be found, following Eq. (3) of Carslaw and Jaeger (1959, p. 267) and by integrating Eq. (21), to be

$$\theta_{as} = \frac{Q_2}{2\pi D} \exp\left[\frac{(K_2 + Vf_2)z}{2D}\right] \times K_0\left[\frac{(K_2 + Vf_2)(x^2 + z^2)^{1/2}}{2D}\right] \quad (22)$$

where

$K_0[\ ]$ is the modified Bessel function of the second kind of order zero.

Both Eqs. (21) and (22) offer very useful means to assess the extent of irrigation affected by the concurrent water and air flow in the soil, or the zone wetted by advancing water.

Compared to previous investigations by others such as Philip (1969, p. 258) who presented a steady-state solution to single phase, one-dimensional linearised Richards equation, the present analysis provides a tool for exploring more information and details regarding two-phase flow in unsaturated soils.

4. Graphic illustration of the results

In this section, we depict Eqs. (21) and (22) and examine some of their major features by analysing the effects of key parameters appearing in the model. Both two- and three-dimensional cases are examined.

4.1. Three-dimensional case

4.1.1. The effect of delivery rates on the geometry of the soil water profiles

With Eq. (21), we graphically illustrate the effect of flow rates in the envelopes formed by steady-state soil water profiles based on data from both the field and literature. We illustrate the flow from a single emitter as a point source in Fig. 1.

Note that in this paper we define the positive direction downwards, as such the minus sign in the vertical coordinate implies the height above the point emitter in three dimensions, and the line source emitter in two dimensions.

As expected it can be seen from Fig. 1 that the soil water profiles formed under a large flow rate extends further than the one formed under a small rate.

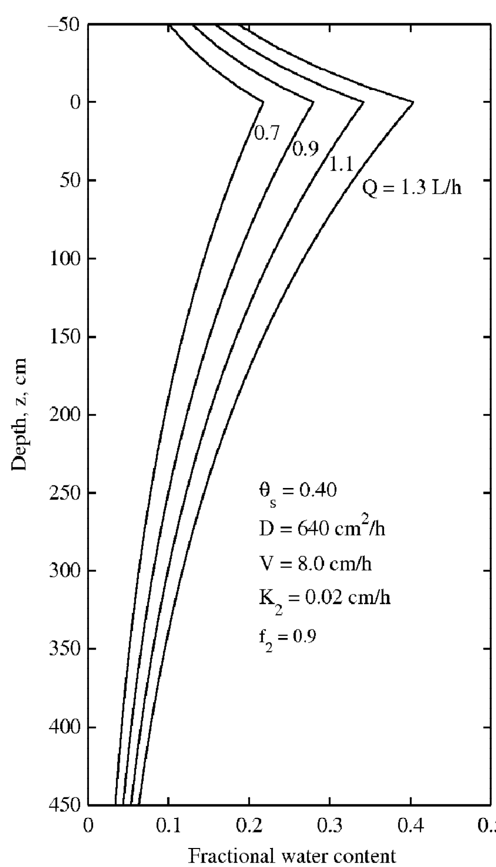

Fig. 1. The effect of flow rate on water content profiles in 3D.

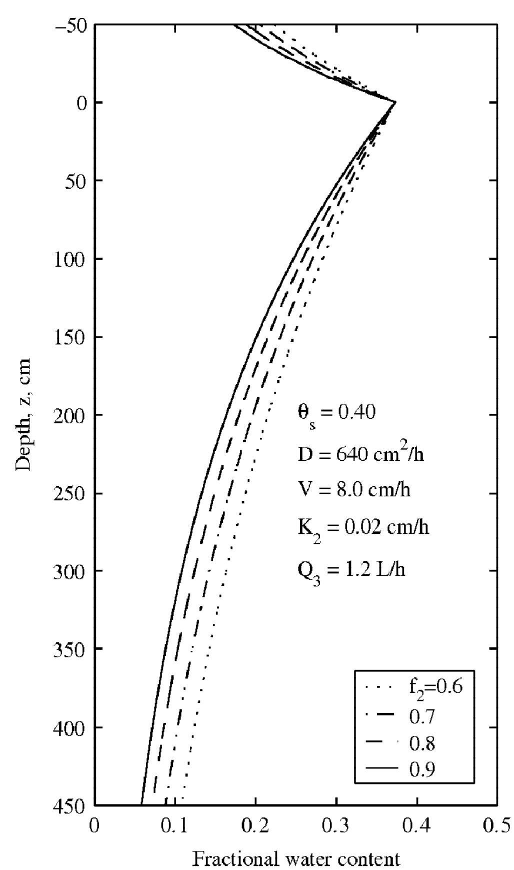

Fig. 2. The effect of parameter f_2 (fractional flow function, f_w) on flow patterns in 3D.

The effect of gravity on flow patterns is significant as seen on the asymmetrical curves.

4.1.2. The effect of parameter f_2 on fractional flow function, f_w

As seen from Eq. (19), the two parameters appearing in the fractional flow function represent the shapes of the fractional flow which are determined from soil and flow properties. Fig. 2 illustrates the effects of f_2 (therefore, the fractional flow function f_w) on flow patterns.

As it can be seen from Fig. 2 that the decrease in f_2 indicates the increase in pores available for airflow in the soil which also facilitates water flow. It also implies that as f_2 decreases the soil water profiles move further both horizontally and vertically. This phenomenon further suggests that a higher proportion of the gas (oxygen or air) in the irrigation water would be more beneficial than a lower one.

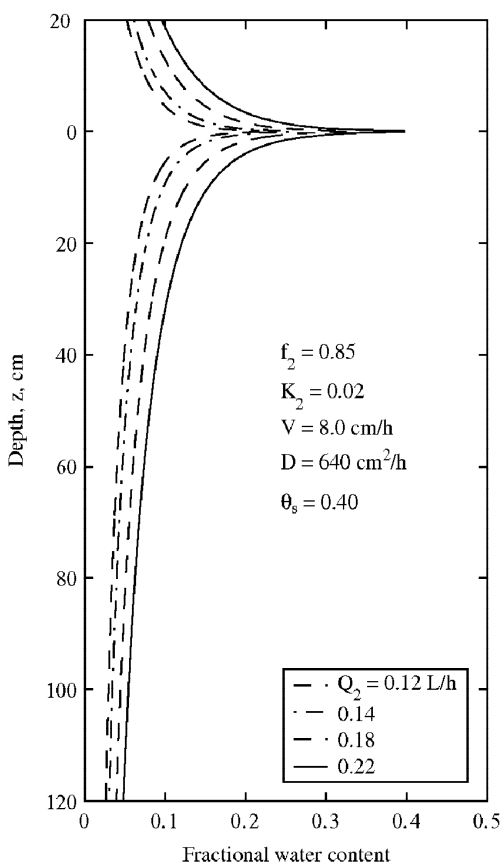

Fig. 3. The effect of flow rate on water content profiles in 2D.

4.2. Two-dimensional case

4.2.1. The effect of delivery rate

The two-dimensional (2D) solution corresponds to a line source. The mathematical form of Eq. (22) for the 2D steady state flow of water looks more complicated than Eq. (21) for the three-dimensional flow. However, the flow patterns revealed by the two forms of solutions are similar as shown in Fig. 3.

It can be seen that Figs. 1 and 3 follow the similar trends.

4.2.2. The effect of fractional flow function (aeration)

Similar to the effect of gas fraction in the three-dimensional case as in Fig. 2, a line source supply and resultant water spreading is shown in Fig. 4.

It is also seen that Figs. 2 and 4 follow similar trends as f_2 changes.

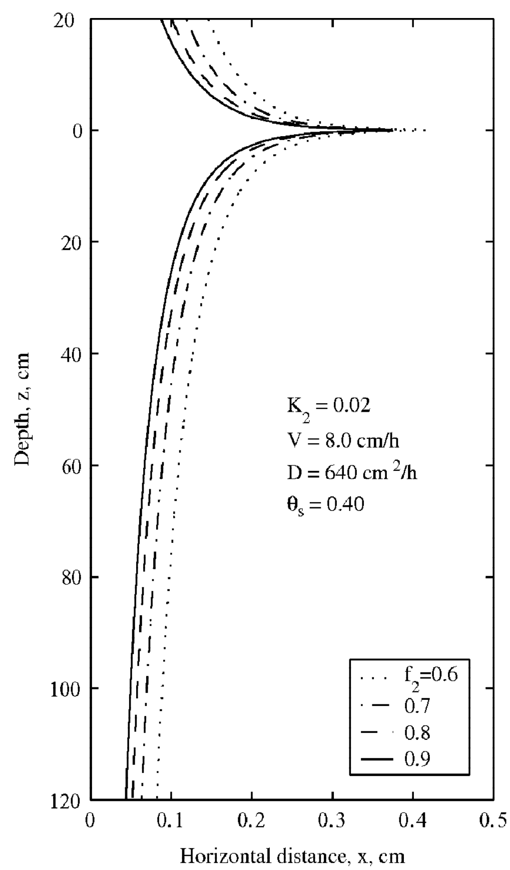

Fig. 4. The effect of parameter f_2 (fractional flow function, f_w) on flow patterns in 2D.

In addition to the geometrical similarities and differences in the two flow patterns, the practical implication of the above analysis is that the height of diffused water above the supply source (either point or line) can be used as an important criterion for designing subsurface drip irrigation schemes, because this depth (height in the figures) represents a potential domain in which potential soil evaporation could develop in a dry climate.

5. Concluding remarks and discussion

In the preceding analysis, we have applied the two-phase flow theory to the aerated irrigation, named oxygation. The following highlights our major contributions in this paper:

1. The original flow equation for concurrent flow of air and water developed by McWhorter (1971) is one-dimensional only, we have extended it to three dimensions (3D) for the analysis of subsurface drip irrigation which reflects more realistic processes in the nature. With this extension, one can analyse airflow-affected water flow in the soil during subsurface drip irrigation which can be regarded as a point source. To our knowledge, the present analysis is the first report on a three-dimensional McWhorter's two-phase flow equation applied to aerated irrigation.
2. With linear functions for both the unsaturated hydraulic conductivity, $K(\theta)$ as suggested by others such as Parlange and Fleming (1984), the fractional flow function, f_w, and a constant diffusivity, D, the 3D equation of concurrent flow of air and water is reduced to the linearised advection–dispersion equation, or Fokker–Planck equation for which standard solutions are readily available for a number of problems (Carslaw and Jaeger, 1959). We have presented exact solutions for both steady- and unsteady-state 3D and 2D McWhorter equation. The 2D solution appears as the modified Bessel function of the second kind of order zero, which corresponds to a line source (supply) while the 3D solution is a complex exponential function.

   The steady-state solution corresponds to concurrent flow of air and water established at large time, and is valuable for several purposes including (1) evaluating the advancing front and extent of aerated subsurface irrigation using emitters, and (2) estimating flow parameters.

3. Graphic analyses and illustrations of the mathematical solutions indicate that due to gravity effects there are asymmetrical distributions of water around the supply source at the steady state established at large time, which differ in patterns for 2D and 3D. These features are valuable for aiding in subsurface irrigation design.
4. The above examples have also illustrated some basic features of the two- and three-dimensional flow patterns. More features of the flow patterns can be examined further in terms of variable conductivity, diffusivity, and unsteady-state patterns. As this paper is designed to address the major fundamental properties of two- and three-dimensional flows encountered in aerated subsurface irrigation, or oxygation, we do not intend to address all these details.

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