Hilgardia

A JOURNAL OF AGRICULTURAL SCIENCE PUBLISHED BY
THE CALIFORNIA AGRICULTURAL EXPERIMENT STATION

Volume 60 • Number 1 • February 1992

Feedforward-Feedback Control of a Cross-Flow Grain Dryer

David Platt, Ahmet Palazoglu, and Tom R. Rumsey

UNIVERSITY OF CALIFORNIA DIVISION OF AGRICULTURE AND NATURAL RESOURCES

ABSTRACT

A feedforward-feedback control of a cross-flow dryer is developed based on a simple mass balance on the dryer, rather than on indirect approaches and empirical relations. The feedforward action is based on a pseudo inlet moisture and adjusts the grain flow to correct for inlet moisture variations. A feedback loop is also added to account for possible process changes and model inaccuracies.

Simulations of a rice dryer show good control of the outlet moisture for variations in the inlet moisture with or without simultaneous disturbances in air flowrate. Effect of the feedforward algorithm on feedback control is also investigated, and the suggested combination of feedforward and feedback controls is tested for variations in the inlet moisture when its measurements are biased.

THE AUTHORS:

David Platt is on leave from RAFAEL, Israel.

Ahmet Palazoglu is Associate Professor, Department of Chemical Engineering, University of California, Davis.

Tom R. Rumsey is Associate Professor, Department of Agricultural Engineering, University of California, Davis.

David Platt, Ahmet Palazoğlu, and Tom R. Rumsey

Feedforward-Feedback Control of A Cross-Flow Grain Dryer[1]

INTRODUCTION

IN RECENT YEARS, many advanced techniques have been suggested to control the grain drying process, as reviewed by Gui et al. (1988). The controlled variable is generally grain moisture at the outlet, and the manipulated variable is grain flowrate. In the past, in the absence of on-line moisture sensors, temperature measurements were used as controlled variables. Each control scheme can be classified into two main categories: feedback and combined feedforward-feedback (use of feedforward alone is uncommon in industrial practice).

The classical feedback control fails to deal effectively with changes in the grain entering the dryer (Whitfield 1986) for the following reasons.

1. There is a large time delay until such changes are sensed at the outlet.
2. The control action is based on the outlet moisture alone, whereas it should take into account the grain presently in the dryer.
3. The sensitivity of the control loop varies with the input moisture content (Marchant 1985) and with the other inputs (inlet temperatures, inlet air humidity, and the grain and air flowrates) due to the nonlinear character of the process.

The instability caused by the nonlinearities of the control loop can be partially overcome by changing the definitions of the controlled and the manipulated variables. They can be redefined, based on the exponential decay-type drying model, as the logarithmic moisture and the residence time of the grain, respectively (Marchant 1985; Whitfield 1988).

Nybrant and Regnér (1985) and Nybrant (1988) used an adaptive self-tuning technique to cope with the varying dynamics of the drying process. They applied the control to a cross-flow dryer based on an empirical linear relation between the exhaust air temperature and the residence time of the grain. The drawback of such a method is that it is indirect, that is, it attempts to control the outlet air temperature rather than the variable of interest, the outlet moisture.

Feedforward control is based on the measurement of the inlet moisture, which is considered to be the most critical load. Unlike the feedback control, feedforward control responds immediately to changes in the inlet moisture and thus prevents the time delay in the correctional action. Model-based feedforward schemes perform better than conventional lead/lag feedforward controllers (Forbes et al. 1984).

A model-based feedforward controller generally uses a simple empirical model with one or two parameters that are adapted to changes in the process dynamics. Forbes et al. (1984) used the exponential decay model as a base for feedforward control of a cross-flow dryer. The same kind of dryer was controlled by Eltigani and Bakker-Arkema

[1]Accepted for publication November, 1991

(1987) who applied a model-based feedforward scheme that uses a linear relation between the outlet moisture and the residence time of the grain. Bakker-Arkema et al. (1989) examined the feedforward control of both cross-flow dryer models. Recently, Moreira and Bakker-Arkema (1990) demonstrated the implementation of a feedforward-feedback adaptive control strategy on a commercial grain dryer.

An important feature in the successful application of a model-based feedforward control is the use of a pseudo inlet moisture, rather than the actual one, in calculating the desired grain flowrate. The pseudo inlet moisture is a function of the initial moisture content of all the grain currently in the dryer. Thus, unlike the feedback control, the control action takes into account all the grain influenced by it.

Although the model-based feedforward control *indirectly* includes a feedback correction (using the currently estimated model parameters in determining the grain flowrate), industrial units should also have conventional feedback based on the error in the outlet moisture. The feedback is required to compensate for possible model errors or unmeasured process disturbances, or both (Stephanopoulos 1984). This study presents a new model-based feedforward controller for cross-flow dryers, which uses a model-based mass balances on the dryer rather than empirical equations. The next section elaborates on the development of the feedforward algorithm. This is followed by a discussion of the algorithm when used with feedback control and the issues of gain scheduling and parameter tuning. Finally, the simulations are presented.

FEEDFORWARD CONTROL ALGORITHM

Development of the Algorithm

The feedforward algorithm is based on a material balance on the water vapor in the air. In a typical cross-flow dryer, the residence time of the air is about 3 orders of magnitude less than that of the grain (Platt et al. 1990). Thus, it may be assumed that a quasi-steady state for air properties exist. As a result, the rate of moisture removal from the grain in the dryer per unit of dryer depth (see fig. 1 for a schematic of a cross-flow dryer) may be given by $G_a L (\overline{W}_{out} - \overline{W}_{in})$.

Suppose that the average inlet moisture ($\overline{M}_{in}$) is time-invariant and that the set point for the outlet moisture (M_{out}) is M_{set}. To achieve the desired value for the outlet moisture, the grain flowrate should obey the equality:

$$G_p = \frac{G_a L (\overline{W}_{out} - \overline{W}_{in})}{b (\overline{M}_{in} - M_{set})} . \qquad [1]$$

By definition, the residence time of the grain is related to its mass velocity by

$$\tau = \rho (1-\epsilon) L/G_p . \qquad [2]$$

If one substitutes G_p from Eq. 1 into Eq. 2, the set point for the grain residence time for the control algorithm is obtained by

$$\tau_F = \frac{\rho (1-\epsilon) b (\overline{M}_{in} - M_{set})}{G_a (\overline{W}_{out} - \overline{W}_{in})} . \qquad [3]$$

Since $\overline{M}_{in}$ is typically not constant, a *pseudo inlet moisture* is defined, which takes into account the inlet moistures of the grain presently in the dryer. The arithmetic average of these inlet moistures will serve as the pseudo inlet moisture. The justification for such a choice is the almost uniform drying efficiency of the dryer along its length. Platt et al. (1990) showed that the steady-state moisture profiles in a cross-flow dryer along the y direction (see fig. 1) for constant x are almost linear. This means that each transversal section of the dryer has a similar drying efficiency. The reason for this phenomenon lies in the very moderate changes in the rate of drying along the dryer as shown by the drying rate profiles in the above-mentioned study. Therefore, in determining the set point for the grain flowrate, there seems to be no reason to give a larger weight to the grain near the top of dryer than that given to the grain near the

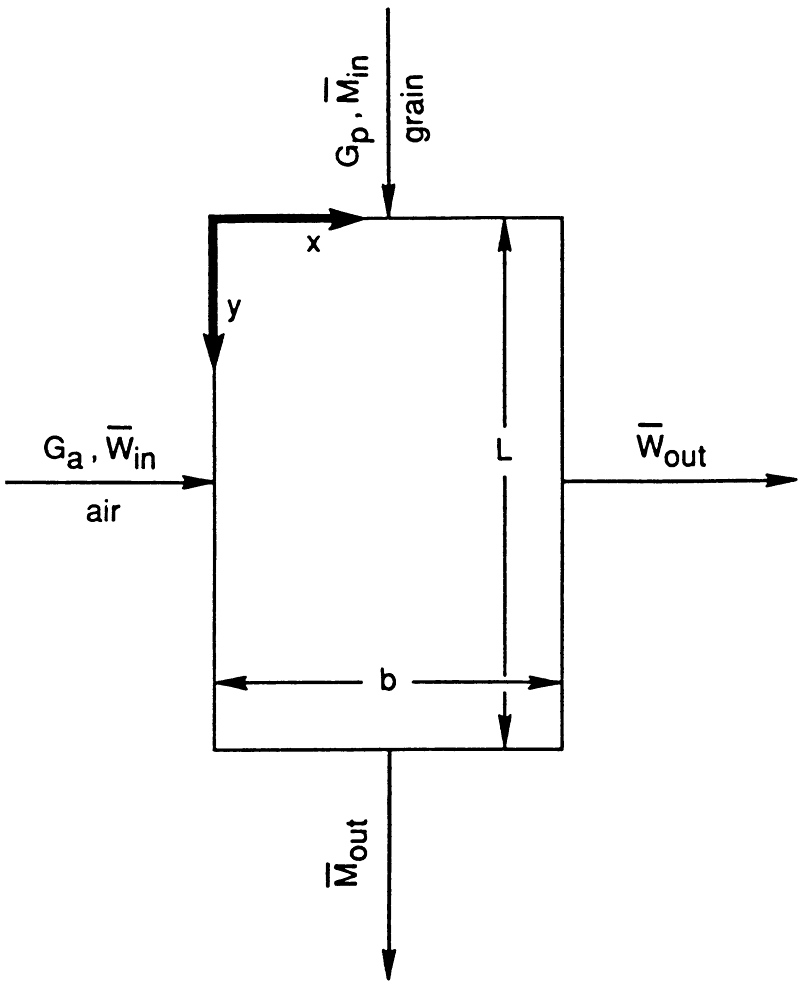

Fig. 1. A scheme of a cross-flow dryer.

bottom. Notably, this conclusion is in contrast with the comment made by Eltigani and Bakker-Arkema (1987). Thus, in the case where the inlet moisture changes with time, $\overline{M}_{in}$ in Eq. 3 is replaced by $\overline{\overline{M}}_{in}$, the arithmetic average of the initial moisture contents of the grain currently in the dryer is

$$\tau_F = \frac{\rho\,(1-\epsilon)\,b\,(\overline{\overline{M}}_{in} - M_{set})}{G_a\,(\overline{W}_{out} - \overline{W}_{in})}\,. \qquad [4]$$

The manipulation of the grain flowrate according to Eq. 4 takes into consideration changes in the input air as well as changes in the input grain. Therefore, the grain temperature in the dryer can be controlled by manipulating the inlet air temperature (a common practice in the industry) without affecting the effectiveness of the outlet moisture control.

Implementation of the Control Algorithm

Implementation of the control algorithm (either the feedforward or the combined feedforward-feedback) requires calculation of quantities that depend on the history of the grain presently in the dryer and of the grain that has just left the dryer. This means that each layer of grain should be tracked from entering the dryer until leaving it.

The dryer is conceptually divided into N_R rows of equal length (see fig. 2). The grain content corresponding to each row is assumed to retain its identity while moving through the dryer and not to mix with the grain in the other rows. Thus, in the simulations, N_R is identical to the number of rows chosen for the numerical solution of the dryer model (see the Appendix). However, in practical implementation of the control algorithm, N_R can be chosen to be much larger.

The control action is not updated continuously but at discrete points of time t_j ($j = 1, 2,\ldots$). Between t_j and t_{j+1}, during the j^{th} time interval, the grain flowrate remains constant, corresponding to a constant residence time $\tau(j)$. t_{j+1} is the time at which the grain content of the i^{th} row at $t{=}t_j$ occupies the $i{+}1^{th}$ row. The relation between t_{j+1} and t_j, which determines the duration of the j^{th} time interval, is given by

$$t_{j+1} = t_j + \tau(j)\,/\,N_R\,. \qquad [5]$$

A dimensionless residence time is defined by

$$\tau^* = \tau/\tau(0)\,. \qquad [6]$$

Then, Eq. 5 can be written in dimensionless form as

$$t_{j+1}^* = t_j^* + \tau^*(j)\,/\,N_R\,. \qquad [7]$$

The grain flowrate during the j^{th} interval becomes

$$G_p^*(j) = 1/\tau^*(j)\,. \qquad [8]$$

With $\tau^*(j)$ evaluated at $t^* = t_j^*$, the grain flowrate for the j^{th} time interval is found from Eq. 8, and t_{j+1}^*, the next time at which τ^* should be reevaluated, is given by Eq. 7. $\tau^*(j)$ is evaluated at each time $t^* = t_j^*$, according to the control algorithm chosen.

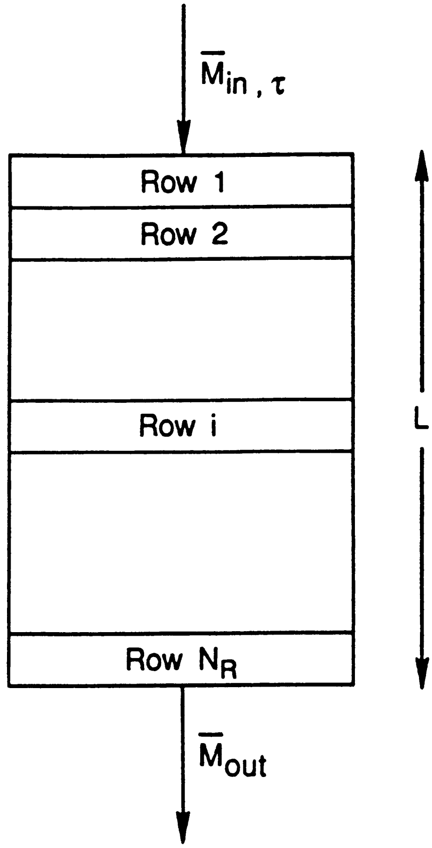

Fig. 2. Division of the dryer into rows.

Implementation of Feedforward Control

To implement the feedforward control, the residence time of the grain is evaluated according to Eq. 4. Assuming ρ, ϵ, b, and M_{set} are constant, all the other quantities of the right-hand side of Eq. 4, $\overline{M}_{in}$, $\overline{W}_{out}$, $\overline{W}_{in}$, and G_a, generally vary with time. For discrete points of time, $t=t_j (j=1, 2, \ldots)$, Eq. 4 yields

$$\tau_F(j) = \frac{\rho\,(1-\epsilon) b \left(\overline{\overline{M}}_{in}(j) - M_{set}\right)}{G_a(j)\left(\overline{W}_{out}(j) - \overline{W}_{in}(j)\right)} . \qquad [9]$$

In an actual dryer, $\overline{W}_{out}(j)$, $\overline{W}_{in}(j)$, and $G_a(j)$ are to be measured at $t = t_j$, whereas $\overline{\overline{M}}_{in}(j)$ is a result of a calculation based on measurements taken at and before $t = t_j$.

Eq. 9 is converted into a dimensionless form (see Appendix)

$$\tau_F^*(j) = \frac{C\left(\overline{\overline{M}}_{in}^*(j) - M_{set}^*\right)}{G_a^*(j)\left(\overline{W}_{out}^*(j) - \overline{W}_{in}^*(j)\right)} \qquad [10a]$$

where C is a dimensionless constant given by

$$C = \frac{G_p(0)\, b\, \overline{M}_{in}(0)}{G_a(0)\, L\, \overline{W}_{in}(0)} . \qquad [10b]$$

To calculate $\overline{\overline{M}}_{in}*(j)$, define $\overline{M}_{in}*\ (i,j)$ as the average dimensionless inlet moisture of the grain, occupying the i^{th} row of the dryer at the j^{th} time interval $(t* = t_j*)$. Assuming we start from a steady state with the inlet moisture equal to $\overline{M}_{in}(0)$, for the first time interval

$$\overline{\overline{M}}_{in}*(1) = 1 \qquad [11a]$$

is obtained. Realizing that a change in the dryer content between $t* = t_{j-1}*$ and $t* = t_j*$ is only due to the grain entering and leaving the dryer during the j-1[th] time interval, $\overline{\overline{M}}_{in}*(j)$ may be expressed in terms of $\overline{\overline{M}}_{in}*(j-1)$ for $j \geq 2$ as

$$\overline{\overline{M}}_{in}*(j) = \overline{\overline{M}}_{in}*(j-1) + (\overline{M}_{in}*(1,j) - \overline{M}_{in}*(N_R,j-1) / N_R . \qquad [11b]$$

$\overline{M}_{in}(1,j)$ is identical to the average dimensionless inlet moisture of the grain entering the dryer during the j-1[th] time interval $(j \geq 2)$

$$\overline{M}_{in}*\ (1,j) = \frac{\int_{t*_{j-1}}^{t_j*} \overline{M}_{in}*\, dt*}{t_j* - t_{j-1}*} = \frac{\int_0^{t_j*} \overline{M}_{in}*\, dt* - \int_0^{t_{j-1}*} \overline{M}_{in}*\, dt*}{\tau*\ (j-1) / N_R} . \qquad [12]$$

The calculation of $\overline{M}_{in}*(1,j)$ requires an on-line measurement of $\overline{M}_{in}$.

$\overline{M}_{in}*(N_R, j-1)$, at the right-hand side of Eq. 11b, can be expressed as

$$\overline{M}_{in}*(1,1) = 1 , \qquad [13a]$$

$$\overline{M}_{in}*\ (N_R, j-1) = \overline{M}_{in}*\ (1, j-N_R) \text{ for } j > N_R , \qquad [13b]$$

and

$$\overline{M}_{in}*\ (N_R, j-1) = 1 \text{ for } j \leq N_R .$$

Practically, $\overline{M}_{in}*(N_R,j-1)$ is found by shifting the data along the rows of the dryer at the end of each time interval

$$\overline{M}_{in}*\ (i+1, j+1) = \overline{M}_{in}*\ (i, j), \text{ for } 1 \leq i < N_R \qquad [14]$$

where $\overline{M}_{in}*(1,j)$ is given by Eqs. 12 and 13a.

Thus, $\tau_F*(j)$ in Eq. 10a can be evaluated, and the grain flowrate can be manipulated accordingly.

FEEDBACK-FEEDFORWARD CONTROL

Feedback control is an essential element in industrial units, because unlike the feedforward control, it is based directly on the error in the outlet moisture, whose minimization is the goal of any control scheme. The feedback is expected to correct for the control actions in a feedforward algorithm for incorrect measurements or for variations in process conditions.

The combined feedforward-feedback algorithm expresses the residence time of the grain during the j^{th} time interval as a sum of two terms, one corresponding to the feedforward, and the other to the feedback

$$\tau^*(j) = \tau_F^*(j) + \tau_B^*(j) \qquad [15]$$

where $\tau_F^*(j)$ is given by Eq. 10a and $\tau_B^*(j)$ is the *correctional* feedback.

The reason for using the residence time of the grain, rather than grain flowrate as the manipulated variable, is the approximate linear dependence of the outlet moisture on the grain residence time (Holtman and Zachariah 1969; Eltigani and Bakker-Arkema 1987; Bakker-Arkema et al. 1989). Note that the choice of the manipulated variable between the grain residence time and its flowrate is not critical for feedforward control, but in feedback control this choice improves the stability margin by reducing the impact of nonlinearities (Marchant 1985; Whitfield 1988).

The feedback implemented in this study is based on the classical proportional-integral (PI) algorithm (Stephanopoulos 1984) with two modifications:

1. The error is defined in order to compensate for the error contributed by the feedforward algorithm.
2. The feedback gain is updated in the beginning of each time interval according to the average inlet moisture of the dryer grain content.

Therefore, the feedback contribution τ_B^* to the total residence time τ^* is given by

$$\tau_B^*(j) = K_B(j)\left(e^*(j) + r\, I^*(j)\right) \qquad [16]$$

where $I^*(j)$ is the integral of the dimensionless error

$$I^*(j) = \int_0^{t_j^*} e^*(u)\,du \cong \sum_{k=2}^{j} e^*(k)\left(t_k^* - t_{k-1}^*\right) \qquad [17]$$

$$= \left(\sum_{k=2}^{j} e^*(k)\,\tau^*(k-1)\right) / N_R\,.$$

$e^*(k)$ is the dimensionless error of the feedback loop that will be defined shortly. According to Eq. 17, $I^*(j)$ can be approximated in terms of $I^*(j-1)$

$$I^*(j) \cong I^*(j-1) + e^*(j)\,\tau^*(j-1)/N_R \qquad [18a]$$

$$\text{for } j = 2, 3, \ldots$$

where

$$I^*(1) = 0\,. \qquad [18b]$$

Unlike $\tau_F^*(j)$, $\tau^*_B(j)$ may accept negative values. This can lead to unrealistic values for $\tau^*(j)$ in Eq. 15, resulting in negative or very high grain flowrates. Thus, a lower bound should be set on $\tau^*(j)$ based on the highest allowable capacity of the dryer unload mechanism. An upper bound to $\tau^*(j)$ should be set as well. Although actual values of this variable will be specific to a dryer operation, large $\tau^*(j)$ generally implies long time intervals within which the control action is not updated. This can lead to positive feedback and, thus, instability.

An alternative way to manage large $\tau^*(j)$ was suggested by Zachariah and Isaacs (1966); they increased the number of the dryer rows (N_R) by a factor of 10 when the grain discharge rate was reduced below a minimum level. Thus, the control time interval, given by $\tau^*(j)/N_R$, was maintained within a reasonable range.

The correctional feedback $\tau_B^*(j)$ is a function of the error in the outlet moisture in the beginning of the j^{th} time interval. However, using feedback in conjunction with the feedforward algorithm requires a modification in the usual definition of the error, because part of the error in the outlet moisture is contributed by the feedforward algorithm. The feedforward error is caused when a specific grain layer has an inlet moisture different from the average inlet moisture of the dryer's grain content. Using only one manipulated variable in a distributed system, the feedback algorithm is restricted to the average rather than specific layers as the feedforward algorithm does. Thus, the influence of deviations from the average on the feedback algorithm should be neutralized. This is done by compensating for the feedforward contribution to the error. The dimensionless error $e^*(j)$, used to calculate the correctional feedback $\tau_B^*(j)$ for the j^{th} time interval, is given by

$$e^*(j) = \overline{M}_{out}^*(j) - M_{set}^* - e_F^*(j) \text{ for } j \geq 2 \qquad [19a]$$

where $\overline{M}_{out}^*(j)$ represents the average outlet moisture of the grain layer leaving the dryer during the $j-1^{th}$ time interval and $e_F^*(j)$ is the feedforward error associated with that layer. If started from a steady state with the dimensionless outlet moisture at M_{set}^*, for the first time interval

$$e^*(1) = 0 \qquad [19b]$$

is obtained.

For simulation purposes, $\overline{M}_{out}^*(j)$ is equal to the average dimensionless moisture of the grain occupying the last row of the dryer ($i = N_R$) at the end of the $j-1^{th}$ time interval (just before it leaves the dryer). In reality, the value of $\overline{M}_{out}^*(j)$ is found by integrating the measurement of $\overline{M}_{out}^*$ over the $j-1^{th}$ time interval. Similar to the derivation of $\overline{M}_{in}^*(1,j)$ in Eq. 12

$$\overline{M}_{out}^*(j) = \frac{\int_{t_{j-1}^*}^{t_j^*} \overline{M}_{out}^* \, dt^*}{\tau^*(j-1)/N_R} \qquad [20]$$

can be derived.

$e_F^*(j)$ in Eq. 19a is the error in $\overline{M}_{out}^*(j)$ contributed by the feedforward algorithm. With respect to the grain occupying the i^{th} row of the dryer at $t^* = t_j^*$, the desired residence time for the j^{th} time interval according to the feedforward algorithm (similar to Eq. 10a) is given by

$$\tau_F^*(i,j) = \frac{C\left(\overline{M}_{in}^*(i,j) - M_{set}^*\right)}{G_a^*(j)\left(\overline{W}_{out}^*(i,j) - \overline{W}_{in}^*(i,j)\right)} \qquad [21]$$

where $\overline{W}_{out}^*(i,j)$ and $\overline{W}_{in}^*(i,j)$ refer to the average air humidities in the beginning of the j^{th} time interval at the inlet and the outlet of the i^{th} row, respectively. If changes in $\tau_F^*(i,j)$ are expressed by $\Delta\tau_F^*(i,j)$ while keeping $\overline{M}_{in}^*(i,j)$, $G_a^*(j)$, $\overline{W}_{out}^*(i,j)$, and $\overline{W}_{in}^*(i,j)$ constant, the corresponding change in M_{set}^* is given by

$$\Delta M_{set}^* = -\frac{G_a^*(j)\left(\overline{W}_{out}^*(i,j) - \overline{W}_{in}^*(i,j)\right)\Delta\tau_F^*(i,j)}{C}. \qquad [22]$$

Using $\tau_F^*(j)$ rather than $\tau_F^*(i,j)$ as the residence time for the j^{th} time interval causes the grain occupying the i^{th} row at $t^* = t_j^*$ to have an error $e_F^*(i,j)$ in the average outlet moisture. $e_F^*(i,j)$ can be approximated by assuming that $\overline{W}_{out}^*(i,j)$ and $\overline{W}_{in}^*(i,j)$ are identical to $\overline{W}_{out}^*(j)$ and $\overline{W}_{in}^*(j)$, respectively. During the j^{th} time interval, the grain moves only $1/N_R$ in the dryer length; thus, the use of Eqs. 21, 22, and 10a yields the following expression

$$\tilde{e}_F^*(i,j) = \left(\overline{M}_{in}^*(i,j) - \overline{\overline{M}}_{in}^*(j)\right) / N_R . \qquad [23]$$

The overall feedforward error $e_F^*(j)$ of the grain leaving the dryer during the $j-1^{th}$ time interval is the accumulation of N_R individual errors, caused throughout the N_R time intervals, while the above grain moves through the dryer

$$e_F^*(j) = \sum_{i=1}^{N_R} e_F^*\left(i, j-1+i-N_R\right) \qquad [24]$$

$$\cong \left(\sum_{i=2}^{N_R} \overline{M}_{in}^*\left(i, j-1+i-N_R\right) - \overline{\overline{M}}_{in}^*\left(j-1+i-N_R\right)\right) / N_R .$$

Eq. 24 describes the development of the feedforward error with the time intervals while the grain moves from the inlet ($i = 1$) to the outlet ($i = N_R$). As shown by Eq. 24, this error depends on the history of the grain in the whole dryer from the moment the grain enters until it leaves it.

To save computational time, the total feedforward error $\tilde{e}_F^*(j)$ is not calculated independently for each j by Eq. 24. Instead, its calculation is based on the already calculated $\tilde{e}_F^*(j-1)$, the feedforward error for the previous time interval (in a way similar to the calculation of $\overline{\overline{M}}_{in}^*(j)$ in Eq. 11b). For that purpose, $\tilde{e}_F^*(j)$ is expressed in terms of $\tilde{e}_F^*(i+1,j)$. From Eq. 23,

$$\tilde{e}_F^*(i,j) = \tilde{e}_F^*(i+1,j) + \left(\overline{M}_{in}^*(i,j) - \overline{M}_{in}^*(i+1,j)\right)/N_R \qquad [25]$$

$$\text{for } i = 1,2,\ldots, N_R - 1 .$$

The difference $\overline{M}_{in}^*(i,j) - \overline{M}_{in}^*(1+i,j)$ in Eq. 25, for two consecutive layers of grain, remains constant during the $N_R - 1$ time intervals in which the layers pass together through the dryer. This is because each layer has its own specific inlet moisture. For a layer leaving the dryer during the $j-1^{th}$ time interval, the inlet moisture is given by $\overline{M}_{in}^*(N_R, j-1)$ or by $\overline{M}_{in}^*(1, j-N_R)$. In view of the above,

$$\tilde{e}_F^*(j) = \tilde{e}_F^*(j-1) - \overline{M}_{in}^*(N_R, j-2) + \overline{M}_{in}^*(N_R, j-1) + \left(\overline{M}_{in}^*(j-1-N_R) - \overline{M}_{in}^*(j-1)\right) / N_R \quad \text{for } j = 2,3,\dots . \qquad [26a]$$

If started from steady state with the inlet moisture equal to $\overline{M}_{in}(0)$ and the average outlet moisture equal to M_{set}, the undefined quantities at the right-hand side of Eq. 26a become

$$\tilde{e}_F^*(1) = 0 \, , \qquad [26b]$$

$$\overline{M}_{in}^*(N_R, 0) = 1 \, , \qquad [26c]$$

and

$$\overline{M}_{in}^*(k) = 1 \text{ for } k \leqq 0 \, . \qquad [26d]$$

Note that even when N_R is very large, the last term in the right-hand side of Eq. 26a must not be neglected, because this may cause an accumulative error in $\tilde{e}_F^*(j)$.

Estimating the Error of the Feedback Loop

In deriving $\tilde{e}_F^*(i,j)$ in Eq. 23, it is assumed that $\overline{W}_{out}^*(i,j)$ and $\overline{W}_{in}^*(i,j)$ are identical to $\overline{W}_{out}^*(j)$ and $\overline{W}_{in}^*(j)$, respectively. This assumption is correct for approximately constant humidities at the inlet and the outlet of the dryer. Simulations show that if the inlet humidity is constant, the outlet humidity is approximately constant. Although it is especially true for the steady-state operation of a cross-flow dryer, as shown by the humidity profiles presented by Platt et al. (1990), even a 20% step change in the inlet moisture does not change this phenomenon. However, knowing the qualitative effect of the assumption concerning the uniformity of $\overline{W}_{out}^*$ on the calculation of $\tilde{e}_F^*(i,j)$ in Eq. 23, the value of $\tilde{e}_F^*(j)$ approximated in Eq. 26a can be corrected.

Replacing $\overline{W}_{out}^*(i,j)$ in Eq. 21 by $\overline{W}_{out}^*(j)$ causes $\tau_F^*(i,j)$ to be greater for a relatively wet grain in the dryer (for which $\overline{W}_{out}^*(i,j) > \overline{W}_{out}^*(j)$), and to be smaller for a relatively dry grain in the dryer (for which $\overline{W}_{out}^*(i,j) < \overline{W}_{out}^*(j)$). This tends to reduce the absolute value of $e_F^*(i,j)$ relative to the approximation in Eq. 23. Thus, the correct value of $e_F^*(i,j)$ is given by multiplying the right-hand side of Eq. 23 by a coefficient $K_F(i,j) < 1$. Since the humidity at the outlet of the dryer is almost constant, $K_F(i,j)$ for typical dryers is expected to be close to 1 and not to vary significantly as a function of i and j.

In view of the above, the error $e^*(j)$, on which the feedback algorithm is based, is given by

$$e^*(j) = \overline{M}_{out}^*(j) - M_{set}^* - K_F \, \tilde{e}_F^*(j) \, . \qquad [27]$$

$K_F(i,j)$ is considered constant with a value of K_F for all i's and j's depending on the basic operational conditions. Generally, K_F is calculated to be around 0.8. Its choice will be discussed in detail in the next section.

Gain Scheduling

Updating the feedback gain, K_B, in the beginning of the j^{th} time interval according to $\overline{M}_{in}^*(j)$ results in a gain scheduling strategy (Gui et al. 1988). The gain scheduling is implemented to reduce the sensitivity of the feedback control loop to variations in the input moisture (Marchant 1985; Whitfield 1986). For the mathematical presentation of gain scheduling, the steady-state gain of the feedback loop, S_B, is defined. For uniform and time-invariant inlet moisture, it is

$$S_B = \frac{\partial \overline{M}_{out}^*(s.s)}{\partial(e^* + rI^*)} = \frac{\partial \overline{M}_{out}^*(s.s)}{\partial \tau_B^*} \; \frac{\partial \tau_B^*}{\partial(e^* + rI^*)} \qquad [28a]$$

$$= K_B \frac{\partial \overline{M}_{out}^*(s.s)}{\partial \tau^*} .$$

$(e^* + rI^*)$ is the driving force for a change in the outlet moisture due to the feedback loop, and S_B is the steady-state gain of the outlet moisture relative to that driving force. If the inlet moisture is time-varying, the grain layers in the dryer do not necessarily have the same inlet moisture. In this case, the steady-state gain of the feedback loop is defined by

$$S_B = \sum_{i=1}^{N_R} (S_B)_i \, / \, N_R \qquad [28b]$$

where $(S_B)_i$ is calculated by Eq. 28a for the inlet moisture equal to that of the grain layer occupying the i^{th} row of the dryer. From Eqs. 28a and 28b, S_B may be expressed by

$$S_B = \frac{K_B}{N_R} \sum_{i=1}^{N_R} \left(\frac{\partial \overline{M}_{out}^*(s.s)}{\partial \tau^*} \right)_i . \qquad [29]$$

The aim of the gain scheduling scheme can be phrased mathematically as keeping S_B constant, regardless of the initial moisture content of the grain layers presently in the dryer.

To apply the gain scheduling, the sum of the steady-state gains on the right-hand side of Eq. 29 has to be evaluated. Simulations presented later show that the steady-state outlet moisture may be approximated by

$$\overline{M}_{out}^*(s.s) = M_{in}^* - A \, M_{in}^* \, \tau^* \qquad [30]$$

where A is a dimensionless coefficient depending on M_{in}^*. However, the same simulations indicate that if changes in M_{in}^* are limited to 20%, A does not change considerably and may be assumed constant. Note that Eq. 30 constitutes a generalization of the linear relation between the outlet moisture and the grain residence time proposed by Holtman and Zachariah (1969).

Differentiating Eq. 30 with respect to τ^* and substituting the derivative of $\overline{M}_{out}$(s.s.) into Eq. 29 yields

$$S_B = - K_B \, A \, \overline{\overline{M}}_{in}^* . \qquad [31]$$

Choosing K_B as:

$$K_B = \frac{K_B'}{\overline{\overline{M}}_{in}^*} \quad [32]$$

where K_B' is a constant, keeps the steady-state gain of the feedback loop constant as desired. Thus, the gain scheduling is accomplished by determining $K_B(j)$ $(j = 1, 2, \ldots)$ in the beginning of the j^{th} time interval by $\overline{\overline{M}}_{in}^*(j)$, according to Eq. 32. The constant K_B' is the feedback control gain to be used when the average inlet moisture of the dryer content is equal to that of the initial steady state. K_B' is found by tuning.

According to Eq. 32, the gain scheduling is based on the assumption that the loads, except the inlet moisture, are time-invariant. However, a sensitivity analysis presented in the next section shows that the inlet moisture is one order of magnitude more significant than the other loads. Thus, in most practical situations where the dimensionless loads do not change more than several tenths from their regular values, for design purposes, Eq. 32 may serve satisfactorily for gain scheduling.

NUMERICAL RESULTS

The proposed feedforward and feedforward/feedback algorithms are tested by computer simulations. Details of the cross-flow dryer model equations and their solution are given in the Appendix. The data for the dryer dimensions and the operational conditions at the initial steady state are for a commercial dryer used for experiments by personnel from the Agricultural Engineering Department at the University of California, Davis (see table 1). The properties of the grain and the air at the inlets are assumed to be uniform. The same dryer is used by Platt et al. (1990) to test the dynamic model.

TABLE 1. THE DRYER DATA

Dryer dimensions and bed properties an inverter at $y^* = 0.5$	
$L = 21.3$	$b = 0.28$
$\epsilon = 0.57$	$\rho = 1394$
Initial operating conditions	
$M_{in}(0) = 0.228$	$W_{in}(0) = 0.009$
$\theta_{in}(0) = 36.11$	$T_{in}(0) = 50.83$
$G_p(0) = 8.588$	$G_a(0) = 0.4157$
Calculated parameters	
$R_c = 2.344 \cdot 10^{-5}$	$h(0) = 2.777 \cdot 10^4$
$\alpha = 0.153$	$\gamma = 1.052$
$\delta = 32.866$	$\phi = 17.208$
$\psi = 18.611$	
Dimensionless outputs at the corresponding steady state	
$\overline{M}_{out}(s.s) = 0.9052$	$\overline{W}_{out}^*(s.s) = 1.6523$
$\overline{\theta}_{out}^*(s.s) = 0.1828$	$\overline{T}_{out}^*(s.s) = -0.0326$

Sensitivity Analysis for the Outlet Moisture

To acquire a better understanding of the dynamic simulations presented below, a sensitivity analysis of the outlet moisture is performed by calculating the steady-state gains for the various inputs. In nonlinear processes like drying, the steady-state gains change with the state of the system. To estimate the dimensionless steady-state gains of the dimensionless outlet moisture $\overline{M}_{out}^*$ relative to the dimensionless inputs M_{in}^*, θ_{in}^*, W_{in}^*, T_{in}^*, G_p^*, and G_a^* for the operating conditions of table 1, each of the dimensionless inputs is perturbed by 0.1 and by −0.1, keeping the other inputs at their original values. If two different values for a steady-state gain are obtained, depending on the direction of the input change (positive or negative), it is a sign for process nonlinearity in relation to that input.

Table 2 presents the estimated steady-state gains, revealing the following facts.

- The inlet moisture is the dominant load. Its dimensionless steady-state gain is an order of magnitude larger than those corresponding to the other loads (θ_{in}^*, W_{in}^*, T_{in}^*, and G_a^*).
- The dimensionless steady-state gain corresponding to G_p^* is about 10 times less than that corresponding to M_{in}^*. This means that the change in the grain flowrate to compensate for the influence of the change in the inlet moisture on the outlet moisture is relatively large. Yet, except for the load M_{in}, the dimensionless steady-state gain corresponding to the grain flowrate is several times larger than those corresponding to the other inputs. This is one of the reasons that justifies the use of the grain flowrate as the manipulated variable.
- We concluded that the response of the outlet moisture to changes in M_{in}, θ_{in}, W_{in}, T_{in}, and G_a is approximately linear by examining the influence of the direction of the input change on the steady-state gains. On the other hand, as expected, the response to a change in the grain flowrate is clearly nonlinear.

TABLE 2. STEADY-STATE GAINS OF THE DIMENSIONLESS OUTLET MOISTURE $\overline{M}_{out}^*$ RELATIVE TO THE VARIOUS DIMENSIONLESS INPUTS

Input	Input change	Dimensionless steady-state gain
M_{in}^*	0.1	0.894
	−0.1	0.889
θ_{in}^*	0.1	−0.017
	−0.1	−0.016
W_{in}^*	0.1	0.017
	−0.1	0.018
T_{in}^*	0.1	−0.026
	−0.1	−0.026
G_p^*	0.1	0.085
	−0.1	0.104
G_a^*	0.1	−0.032
	−0.1	−0.034

Feedforward Control

The feedforward algorithm (Eqs. 10a and 10b) is tested for various disturbances, and the results of the simulations are shown in figures 3 to 5. The simulations start from the steady state under the operating conditions of table 1. It is assumed that the outlet moisture for this steady state is the set point: $M_{set}^* = \bar{M}_{out}^*(0) = 0.9052$. The control task is to keep the outlet moisture at its set point despite disturbances in the loads starting at $t^* = 0.2$. All the simulations include a disturbance in the major load, M_{in}. Some of them also include a simultaneous disturbance in another load.

Figure 3 corresponds to a disturbance in M_{in} alone: a 20% step change. Figure 4 has a set of two 20% pulses. The secondary loads considered in our work are θ_{in}^*, W_{in}^*, T_{in}^*, and G_a^*. The heights of the pulses in the secondary loads are chosen to be relatively large (-0.5 for θ_{in}^*, $+1.0$ for W_{in}^*, -0.5 for T_{in}^*, and -0.5 for G_a^*); otherwise, the influence of such disturbances is negligible (according to the sensitivity analysis done above). The large disturbances in the secondary loads may represent sharp changes in the quality of the entering grain or serious disorders in the process, like stops in the operation of an air heating unit or of an air blower.

To prove our point, we present a simulation that is representative of disturbances in the secondary load. Figure 5 shows the controlled responses to a 20% step change in M_{in}^* combined with a set of two pulses in T_{in}^*. Despite the large disturbance in T_{in}^*, comparison of the uncontrolled response in figure 3 with the uncontrolled response in

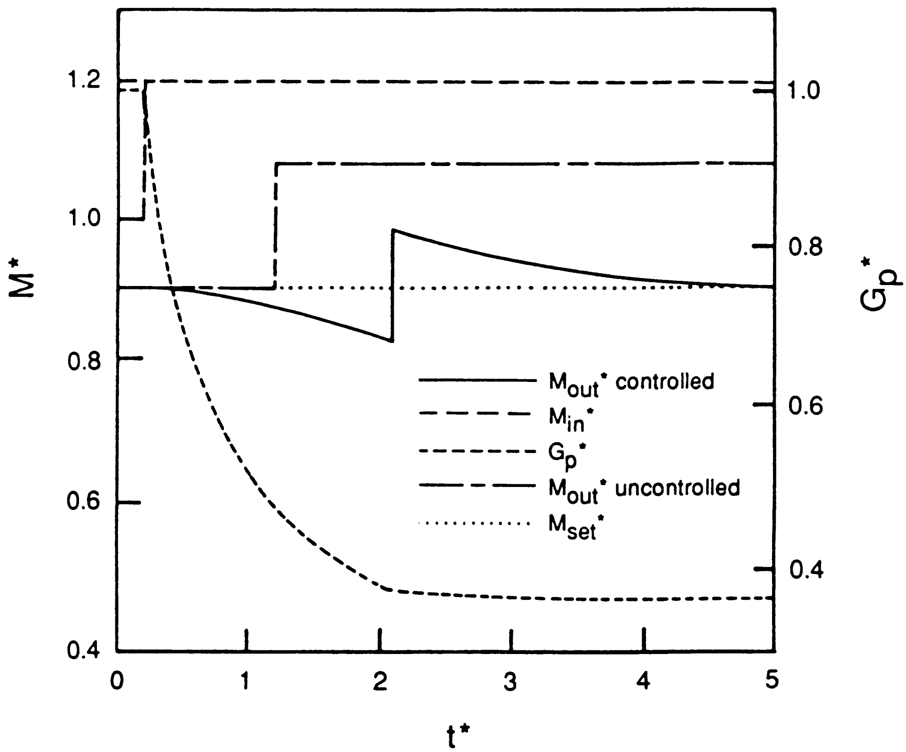

Fig. 3. System response to 20% step change in inlet moisture, feedforward control.

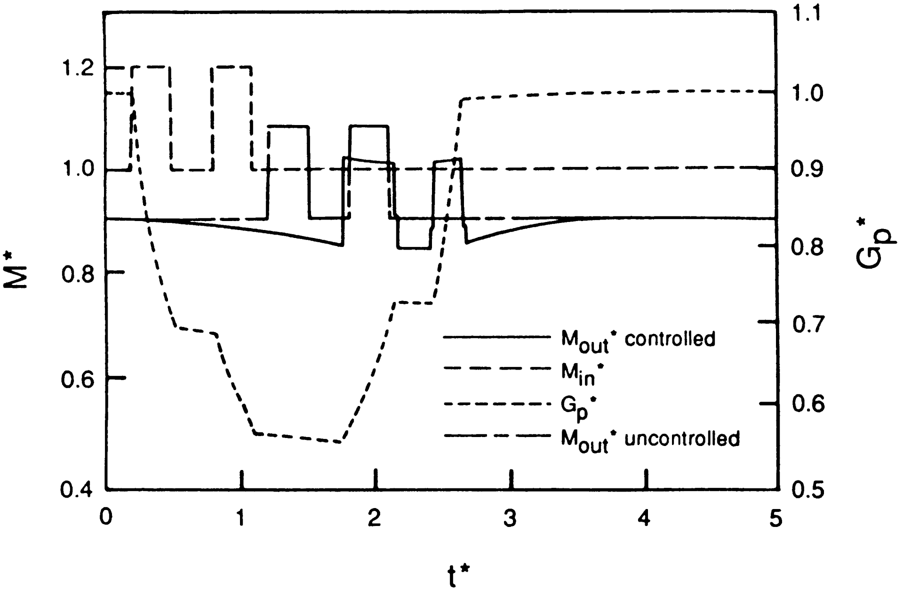

Fig. 4. System response to two 20% pulses in inlet moisture, feedforward control.

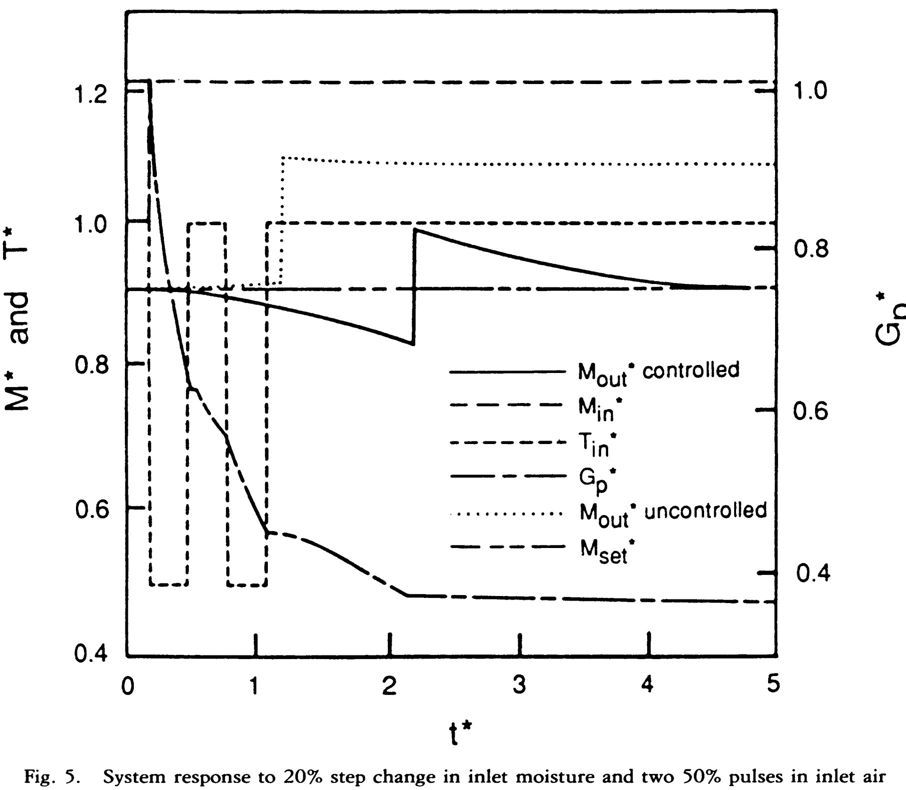

Fig. 5. System response to 20% step change in inlet moisture and two 50% pulses in inlet air temperature, feedforward control.

figure 5 displays a moderate effect of the disturbance in T_{in}^* relative to that of the inlet moisture. We have observed similar trends for other secondary loads.

Remark 1. The curves corresponding to the controlled outlet moisture and to the manipulated grain flowrate in figures 3 to 5 consist of small steps due to the discrete character of the control algorithm. Increasing N_R reduces the control time intervals, causing those curves to be smoother.

Remark 2. Note that each control time interval corresponds to the same amount of grain entering or leaving the dryer, although during the transient period, towards the new steady state, the control time intervals generally differ in their duration (when each time interval is characterized by a step in G_p^* of a different width). If figure 4 is examined, the two pulses in M_{in}^* have the same duration, but the amount of grain corresponding to each in the controlled process is different. The first pulse corresponds to a higher grain flowrate, and therefore involves a greater amount of grain. The dependence of the control time interval on G_p^*, which in turn is time dependent, explains the unequal pulses in the controlled outlet moisture achieved for the equally wide pulses in the inlet moisture.

Remark 3. If we compare the curves for the controlled outlet moisture in figures 3 and 4, we see that both the maximum and minimum values for $\overline{M}_{out}^*$ are higher when the inlet moisture changes in pulses. This is because the relative weight of the wetter grain ($M_{in}^* = 1.2$), in calculating $\overline{\overline{M}}_{in}$ for the pulse disturbances, is smaller compared to that for the step change. This results in a higher average grain flowrate for both the drier grain initially in the dryer ($M_{in}^* = 1$) and the wetter grain entering the dryer due to the disturbance ($M_{in}^* = 1.2$).

Remark 4. Figures 3 to 5 indicate that the feedforward control brings the system to a new steady state with the outlet moisture reaching its set point. Without control, $\overline{\overline{M}}_{out}^*$ in figures 3 and 5 would show a steady-state offset of 0.1795. There is a transient period until the system reaches the desired steady state. The duration of that transient period depends on the type of load.

For disturbances in the inlet moisture, the new steady state is achieved only after two dryer volumes of grain have left the dryer since the inlet moisture stopped changing. This is because the outlet moisture of each grain layer depends on the initial moisture contents of all the grain layers sharing the dryer with that layer while it passes through the dryer (due to the dependence of G_p on $\overline{\overline{M}}_{in}$). For all the secondary loads, the steady state is achieved after only one dryer volume of grain has left the dryer since the last change in the relevant load.

In figures 3 and 5, which correspond to step changes in the inlet moisture, the new steady state is achieved about 4.5 units of dimensionless time after the change in the inlet moisture. The new steady state in figure 4, where M_{in}^* changes in pulses, is achieved earlier (about 2.5 dimensionless time units since the last change in M_{in}^*) due to the higher grain flowrate. In all the controlled processes of figures 3 to 5, the average product moisture during the transient period is maintained close to its set point.

Remark 5. The grain flowrate responds immediately to disturbances in the air (in W_{in}, T_{in}, and G_a), but it responds to disturbances in the grain (in M_{in} and θ_{in}) more smoothly and more gradually. The reason for this is simple. Unlike changes in the air inlet, those in the grain inlet affect the dryer gradually as more grain affected by the disturbance occupies the dryer.

Remark 6. The discontinuities in the curves for the outlet moisture reflect past discontinuities in the inlet moisture. If one focusses on the points of discontinuity for the controlled outlet moisture in the cases of step disturbances in M_{in} (figs. 3 to 5), it can be observed that the deviations of $\overline{M}_{out}{}^*$ from $M_{set}{}^*$ are around 0.085 for the wettest grains and −0.075 for the driest grain. The average of these deviations is just slightly higher than zero, which is the ideal value. A likely reason for that bias is a slight inaccuracy in the feedforward algorithm, caused by the approximation of $\overline{W}_{out}{}^*(i,j)$ by $\overline{W}_{out}{}^*(j)$. This bias affects grain of different moisture contents differently.

Remark 7. The extreme deviations of $\overline{M}_{out}{}^*$ from $M_{set}{}^*$ at the point of discontinuity may aid in estimating K_F in the feedforward/feedback algorithm. Without error sources other than the feedforward algorithm, the feedback error in Eq. 27 is supposed to be zero. The best value for K_F is the one for which the average of $e^*(j)$ ($j = 1,2,\ldots$), using only feedforward control, is close to zero.

For step disturbances in M_{in}, simulations indicate that a good approximation for an acceptable K_F is found by zeroing the average of the feedback errors (Eq. 27) for the two grain layers with the largest deviations from the set point (the layers corresponding to the discontinuity in the curve for $\overline{M}_{out}{}^*$). If the $\overline{M}_{out}{}^*$ curve in figure 3 is taken as an example, the outlet moisture contents corresponding to the point of discontinuity at $t^* = 2.1$ are 0.8296 and 0.9906, deviating from the set point 0.9052 by −0.0756 and 0.0854, respectively. The approximated overall feedforward errors $\tilde{e}_F{}^*(j)$ for the driest and the wettest grain layers are −0.098 and +0.098, respectively. The average of the $e^*(j)$'s for those layers is set to zero by choosing $K_F = 0.82$. The significance of finding an acceptable K_F is elaborated in the next subsection.

Feedforward-Feedback Control

The feedforward-feedback algorithm is tested for a 20% step change in the inlet moisture, whereas the measurement of the inlet moisture is assumed to be biased by a constant value. Due to that bias, $M_{in}{}^*$ for the feedforward control purposes is always 0.05 units below its actual value. If only feedforward control is applied, $\overline{M}_{out}{}^*$ at steady state is higher by 0.0495 units (almost the bias in $M_{in}{}^*$) than its set point (fig. 6). Adding appropriate feedback will delete this offset.

Gain Scheduling. First, Eq. 30, on which the gain scheduling scheme is based, is verified by simulations. This is accomplished by changing the grain flowrate for several inlet moistures, keeping all the other parameters at their values in table 1. The results of the simulations are summarized in table 3. For a better interpretation of the simulation results, Eq. 30 written as

$$1 - \overline{M}_{out}{}^*(s.s) \,/\, M_{in}{}^* = A\,\tau^* \,.$$

Table 3 reveals that the quantity $[1 - \overline{M}_{out}{}^*(s.s) \,/\, M_{in}{}^*]$ changes almost linearly with τ^*, whereas its dependence on $M_{in}{}^*$ is small. This confirms Eq. 30 and allows the assumption of constant A for variations of up to 20% in $M_{in}{}^*$. If larger variations are expected, gain scheduling may still be possible, but it will be computationally more demanding. As a result, the use of Eq. 37 for gain scheduling for such variations in $M_{in}{}^*$ is assumed to be justified.

Determination of K_F. As pointed out earlier, to define the feedback error, the value of K_F has to be evaluated. K_F is found by implementing a step disturbance in the inlet moisture and analyzing the resulting discontinuity in the outlet moisture using feedforward control only (as explained in the former section). The significance of a "good" K_F is seen in figures 7 and 8 for the case where the inlet moisture measure-

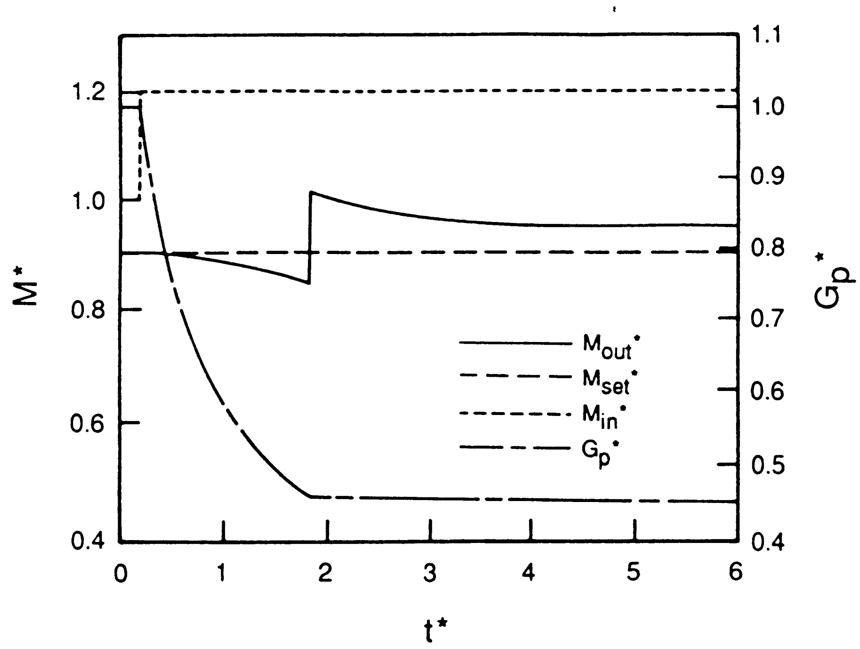

Fig. 6. System response to 20% step change in inlet moisture with bias inlet moisture measurement, feedforward control.

TABLE 3. RESULTS OF SIMULATIONS FOR GAIN SCHEDULING: $1 - \bar{M}_{out}*(s.s.)/M_{in}*$ FOR SEVERAL COMBINATIONS OF $M_{in}*$ and $\tau*$

$\tau*$ / $M_{in}*$	0.667	1	2	4
1	0.0634	0.0948	0.1846	0.3360
1.1	0.0644	0.0958	0.1857	0.3391
1.2	0.0649	0.0961	0.1858	0.3403

ments are not biased. Both figures show the response of $\bar{M}_{out}*$ and G_p* to a step disturbance in $M_{in}*$ under the influence of the feedforward-feedback control, including gain scheduling. The feedback consists of proportional action only with $K_B' = 150$. In figure 7, K_F is chosen to be 0.82, as evaluated in the former section, while figure 8 corresponds to $K_F = 0.91$. The good choice of K_F, as seen in figure 7, is characterized by a smooth curve for G_p, which in turn results in a smooth curve for $\bar{M}_{out}*$. With no sources of error other than the feedforward algorithm, a good choice of K_F should bring the feedback error close to zero at any point of time, which means that the variation range of the feedback error for a good K_F is very limited.

A "bad" choice of K_F leads to erroneous values for the error $e*(j)$ in Eq. 27. In the case shown in figure 8, this causes a sharp change in $e*$ and therefore a sharp change in G_p* at the moment when the inevitable discontinuity in $\bar{M}_{out}*$ is observed ($t* \approx 2.1$).

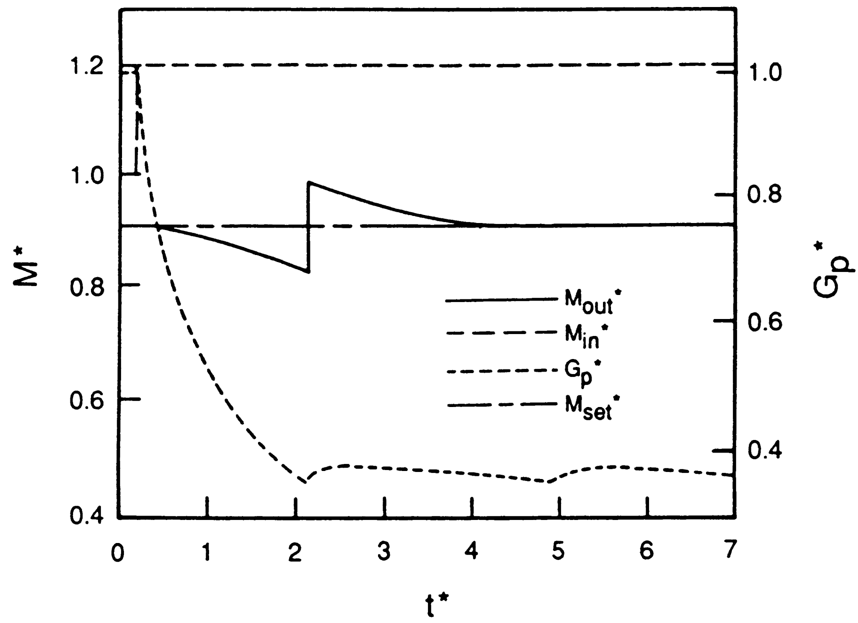

Fig. 7. System response to 20% step change in inlet moisture, feedforward-feedback control with k = 0.82.

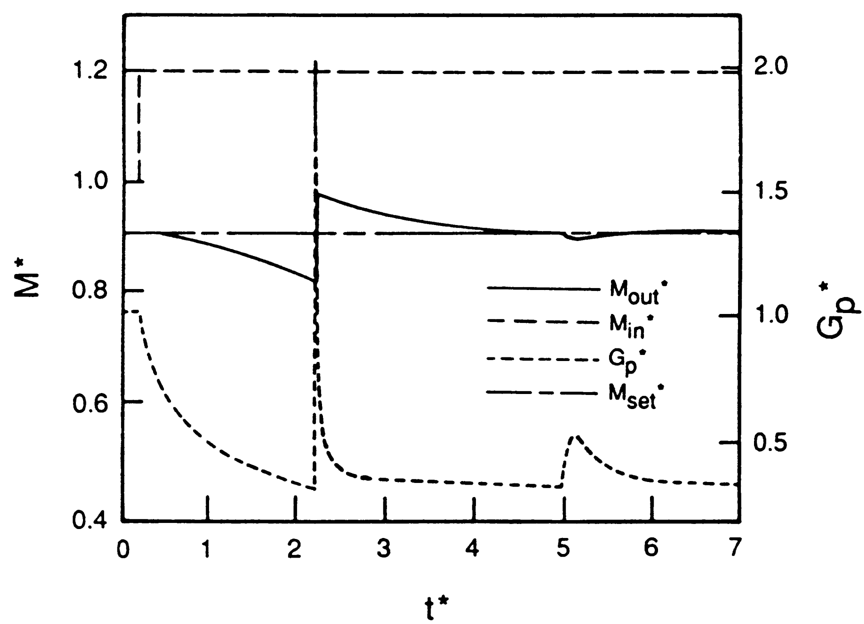

Fig. 8. System response to 20% step change in inlet moisture, feedforward-feedback control with $k_f = 0.91$.

The sharp change in G_p^* induces additional sharp changes in $\overline{M}_{out}^*$ and G_p^* (although much smaller than the first ones) during the settling period.

Despite large oscillations in G_p^* in figure 8, the response of $\overline{M}_{out}^*$ is damped and may be considered satisfactory. G_p^* is much more sensitive to the choice of K_F than $\overline{M}_{out}^*$. For $K_F > 0.91$, large grain flowrates are required by the control algorithm at the moment of the sharp discontinuity in $\overline{M}_{out}^*$. Choosing $K_F = 0.91$ requires $G_p^* = 2.01$ (fig. 8); the maximum G_p^* required for $K_F = 0.92$ is greater than 5. Generally, such a value is higher than the upper limit for G_p^* due to the physical limitations of the unload mechanism. Thus, the control action reaches saturation.

An acceptable choice of K_F is important in case of a sudden change in the inlet moisture, where the feedforward error $e_F^*(j)$ may reach high values. For a gradual change in the inlet moisture, the feedforward error is relatively small, and the influence of K_F on the response is less significant.

Biased Measurement Case. Two extreme cases are examined concerning the biased measurement of the inlet moisture.

1. The measurements of the inlet moisture taken before the occurrence of the disturbance are not biased. The biased measurements start with the step change in the inlet moisture.
2. The measurements of the inlet moisture of the grain in the dryer at the moment of the disturbance ($t^* = 0.2$) are biased as well.

For the control algorithm, the first case is the worst. For some time, the biased measurements affect not only the feedforward algorithm but also the feedback loop. This is because the feedforward error $e_F^*(j)$ is estimated by the sum of terms on the right-hand side of Eq. 23. In each such term, the inlet moisture of a specific layer of grain differs from an average inlet moisture of all the dryer grain content. The above difference is erroneous (1) if the measurements of the inlet moisture before the disturbance are not biased, and (2) whenever a layer of grain for which the measurement of the inlet moisture is biased shares the dryer with another layer for which that measurement is unbiased. This leads to erroneous values of $e_F^*(j)$ and $e^*(j)$ for an initial period after the disturbance. Only after two dryer volumes of grain have left the dryer after the occurrence of the step disturbance will $\overline{M}_{in}^*(i,j)$ and $\overline{\overline{M}}_{in}^*(j)$ in Eq. 23 be biased to the same extent, resulting in an accurate estimation for $e_F^*(j)$ and $e^*(j)$.

In case 2, although $\overline{M}_{in}^*(i,j)$ and $\overline{\overline{M}}_{in}^*(j)$ in Eq. 23 are erroneous, their difference is always correct because they are biased to the same extent. This means that the feedback loop is always based on true values of the error.

The feedforward-feedback algorithm is tested for both cases. At first, the feedback consists of proportional control only with $K_B' = 150$. The responses for cases 1 and 2 are shown in figures 9 and 10, respectively. The initial erroneous e^* for the feedback algorithm in case 1 causes a very sharp change in the grain flowrate at the moment the big discontinuity in $\overline{M}_{out}^*$ is observed ($t^* = 1.9$). This induces later oscillations in G_p^* and $\overline{M}_{out}^*$ showing a slowly dampening response similar to the choice of a bad K_F (fig. 8), which also leads to an erroneous e^*.

Though the response for case 2 (fig. 10) is symmetrical around the set point, the response for case 1 (fig. 9) is biased upward, especially in the initial period after the disturbance. Except for a sharp change in G_p^* at the moment the control is first applied ($t^* = 0.2$), the response in case 2 is much less oscillatory both in G_p^* and $\overline{M}_{out}^*$. The initial oscillation of G_p^* in case 2 occurs because the feedforward in this case is based, from the beginning, on a totally biased value for $\overline{\overline{M}}_{in}^*$ (0.95 instead of 1), unlike case 1

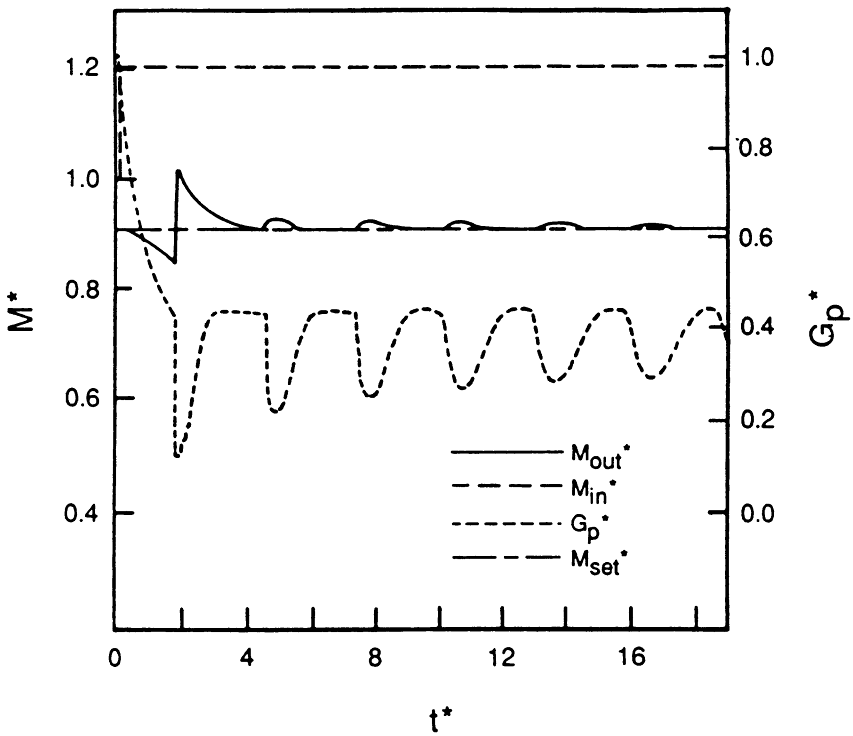

Fig. 9. System response to 20% step change in inlet moisture, biased inlet moisture measurement at step change, feedforward and feedback with proportional control.

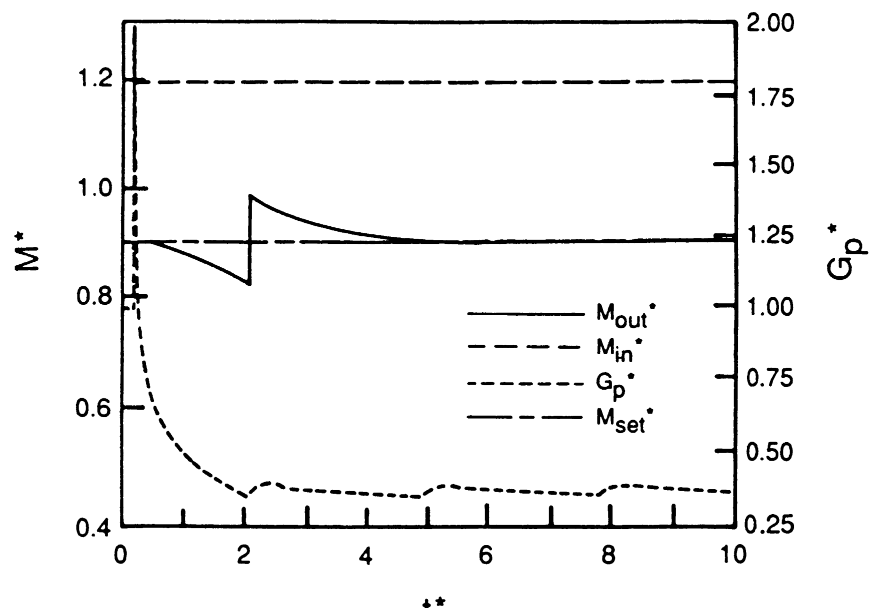

Fig. 10. System response to 20% step change in inlet moisture, biased inlet moisture measurement, feedforward and feedback with proportional control.

in which the value for $\overline{\overline{M}}_{in}$* is initially correct and reaches its maximum bias gradually. However, after a certain initial period, the outlet moisture in both cases shows small oscillations with an average $\overline{M}_{out}$* value about $4 \cdot 10^{-3}$ higher than the set point. Although statistically insignificant, this offset is inevitable using proportional control only because, for a zero error, the feedback is inactive. Eventually, a new error builds itself due to the erroneous feedforward action.

To eliminate this small offset, integral action is added. Adding integral action with r = 0.1 (see figs. 11 and 12 for cases 1 and 2, respectively) does not prevent small oscillations in $\overline{M}_{out}$* but their averages converge to the set point.

Tuning the feedback gain and reset rate (K_B' and r) is not the subject of this work, and the values chosen for them may not be the optimal ones. They are selected to demonstrate the control algorithm. Controller tuning is based on the compromise between increasing speed of response and reducing offset on the one hand and increasing oscillations on the other. Increasing K_B' improves the speed of response and reduces the offset, whereas increasing r contributes mainly to offset elimination. However, increasing either K_B' or r yields a more oscillatory response. Exploration of the optimal tuning parameters is left as future work.

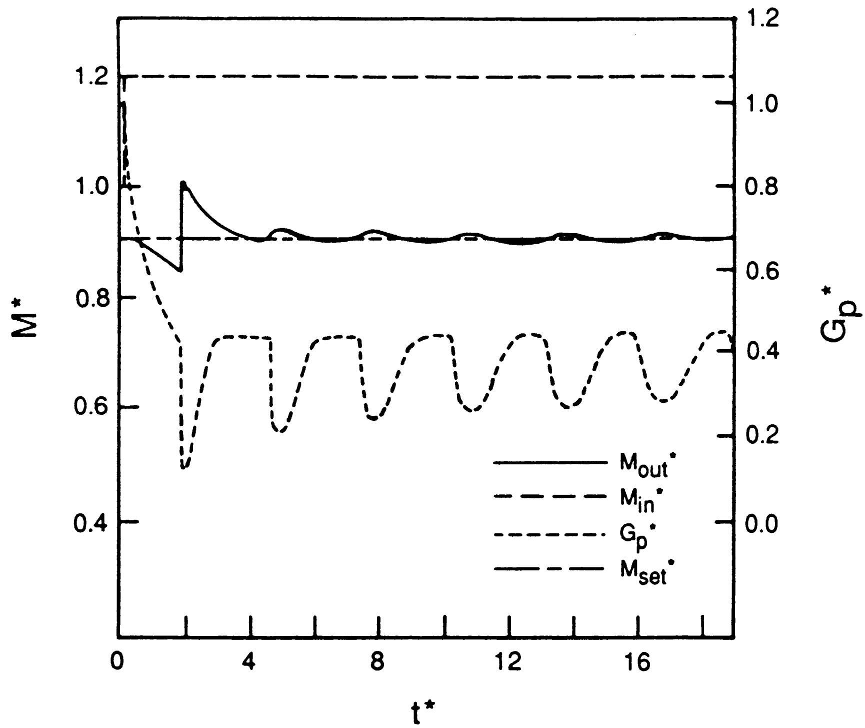

Fig. 11. System response to 20% step change in inlet moisture, biased inlet moisture measurement at step change, feedforward and feedback with proportional plus integral control.

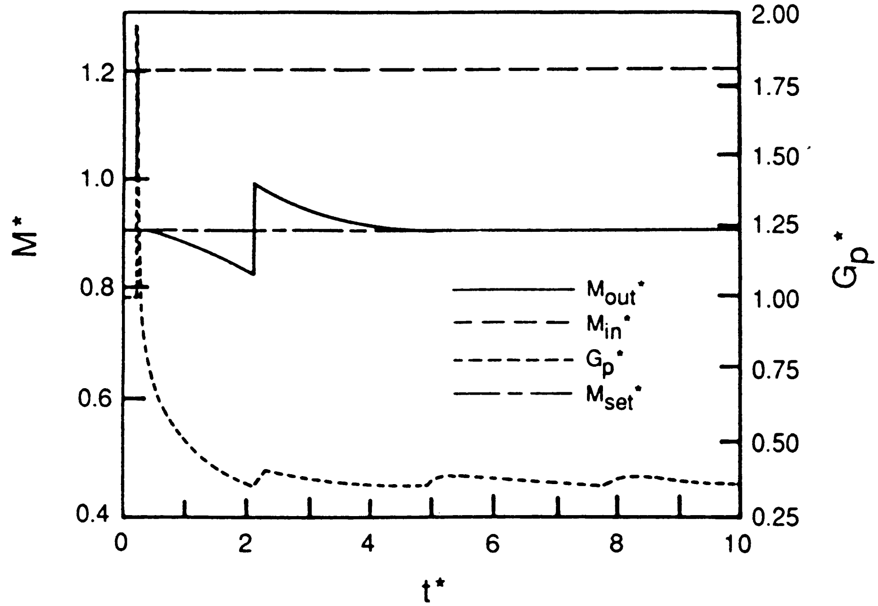

Fig. 12. System response to 20% step change in inlet moisture, biased inlet moisture measurement, feedforward and feedback with proportional plus integral control.

CONCLUSIONS

This study presents a feedforward-feedback algorithm for the control of the grain moisture content in a cross-flow dryer by manipulating the grain flowrate. The feedforward control is based on an instantaneous mass balance on the water vapor in the air rather than on empirical relations. Therefore, feedforward control deals effectively with a variety of disturbances in the loads, which allows for an independent control of the grain temperature without affecting much the effectiveness of the outlet moisture control.

Simulations indicate that the suggested feedforward control scheme successfully handles sharp changes in the inlet moisture even during simultaneous disturbances in air flowrate.

Correctional feedback is added to the feedforward control using a modified definition of the error that compensates for that part of the error inherently generated by the feedforward algorithm. The conventional PI algorithm is used with two modifications, which are required because of nonlinearity of the process. The definition of the manipulative variable is changed from flowrate to residence time, and gain scheduling is applied based on the average inlet moisture of the grain content in the dryer. The combined feedforward-feedback control is tested in simulations for variations in the inlet moisture when its measurements are biased; the feedback loop successfully corrects the erroneous feedforward action based on the biased measurements.

APPENDIX

Cross-flow Dryer Model

The feedforward algorithm (Eq. 4) and the feedforward-feedback scheme are tested by computer simulations for which a model of a cross-flow dryer is needed. The model was previously studied by Platt et al. (1990); and we assume quasi-steady state for the air properties. The mass and energy balances on the dryer in dimensionless form are

$$\frac{\partial M^*}{\partial t^*} = -G_p^* \frac{\partial M^*}{\partial y^*} - \alpha R^* \tag{A1}$$

$$0 = -G_a^* \frac{\partial W^*}{\partial x^*} + \gamma R^* \tag{A2}$$

$$\frac{\partial \theta^*}{\partial t^*} = -G_p^* \frac{\partial \theta^*}{\partial y^*} + \delta h^* (T^* - \theta^*) - \alpha\phi R^* \tag{A3}$$

$$0 = -G_a^* \frac{\partial T^*}{\partial x^*} - \psi h^* (T^* - \theta^*) \tag{A4}$$

where

$$M^* = M/\overline{M}_{in}(0) \tag{A5a}$$

$$W^* = W/\overline{W}_{in}(0) \tag{A5b}$$

$$\theta^* = \frac{\theta - \overline{\theta}_{in}(0)}{T_{in}(0) - \overline{\theta}_{in}(0)} \tag{A5c}$$

$$T^* = \frac{T - \overline{\theta}_{in}(0)}{\overline{T}_{in}(0) - \overline{\theta}_{in}(0)} \tag{A5d}$$

$$t^* = \frac{t}{\tau(0)} = \frac{tG_p(0)}{L\rho(1-\epsilon)} \tag{A5e}$$

$$x^* = x/b \tag{A5f}$$

$$y^* = y/L \tag{A5g}$$

$$G_a^* = G_a/G_a(0) \tag{A5h}$$

$$G_p^* = G_p/G_p(0) \tag{A5i}$$

$$h^* = h/h(0) \tag{A5j}$$

$$R^* = R/R_c\,. \tag{A5k}$$

α, γ, δ, ϕ, and ψ are dimensionless groups (Platt et al. 1990) that depend on the dryer dimensions, initial inlet and operating conditions, and physical properties of the grain, air, and water. The simulations start from a steady state. Thus, the time-dependent dimensionless variables in Eq. 9 are defined by normalization relative to the initial steady-state values.

Solving the Model Equations

To solve the model equations numerically, the dryer is divided into rows and columns, and the compartments thus formed are assumed to be completely mixed. A system of coupled algebraic and ordinary differential equations is obtained by expressing the spatial derivatives by finite differences. The method of solution is detailed in Platt et al. (1990).

The presence of the convection terms in Eqs. A1 and A3 for the grain leads to an effect that can be termed as backmixing (or axial dispersion) when finite difference approximation is used. This effect can be theoretically eliminated if the dryer is divided into an infinite number of rows. Because a finite number of rows is used for simulation purposes, a step change in the inlet will be dispersed and will not appear as a step at the outlet, as it should for this system. For sharp changes in the inlet moisture (e.g., step changes or pulses), this effect should be avoided, otherwise the calculated outlet moisture will show an overdamped response. Therefore, one important modification is made in using the numerical solution method.

The direct way to neutralize this effect in the mathematical model is to omit the convection terms from Eqs. A1 and A3 and to shift the grain content of each row in the dryer at discrete points of time to the next row (in the direction of flow), according to the grain flowrate. This way, grain backmixing in the mathematical model is eliminated and a step change in the inlet moisture leads to a step change in the outlet moisture. This approach was used by Zachariah and Isaacs (1966), Marchant (1985), Nybrant and Regnér (1985), and Nybrant (1986-1988), but they used the technique in conjunction with simple empirical models, assuming a uniform moisture in each row. In this work, the approach is applied to a partial differential equation model, which considers variations of the relevant variables (M, W, T, θ) in the x and the y directions.

An important feature of the above technique is that each grain content occupying a compartment of the dryer keeps its identity during its movement along the dryer. This characteristic of the model is used in calculating factors in the control algorithms related to the grain history presently in the dryer (e.g., $\overline{\overline{M}}_{in}$ in Eq. 4).

Omitting the convection terms from the equations for the grain is an important element in keeping the identity of the grain in the dryer, but is not the only one. Keeping the grain identity dictates a fine division of the dryer into rows, especially when dealing with sharp and large changes in the inlet conditions of the grain. The division into columns may be much less fine. Due to the simplicity of the control algorithm, there is no reason not to divide the dryer into hundreds of rows. However, we chose to divide the dryer into fifty rows and three columns for the purpose of demonstration. *The field application of the algorithm, does not require the numerical solution of Eqs. A1 to A4.* A finer division than 50×3 requires large computing resources. The division of the cross-flow dryer into 50 rows was found by Holtman and Zachariah (1969) to provide sufficient computing accuracy.

NOMENCLATURE

A	—	dimensionless coefficient in Eq. 30
b	—	dryer width (m)
C	—	dimensionless constant defined in Eq. 10b
*e	—	error in the feedback algorithm
e_F	—	error contributed by the feedforward algorithm
$\tilde{e}_F$	—	approximation to e_F
G_a	—	dry air mass velocity, $kg/s - m^2$
G_p	—	dry grain mass velocity, $kg/s - m^2$
h	—	volumetric convective heat transfer coefficient, $J/m^3 - s - {}^\circ C$
i	—	index designating a row in the dryer
I	—	error integral in the feedback algorithm
j	—	index designating time interval
k	—	index designating time interval
K_B	—	dimensionless gain in the feedback algorithm
K_B'	—	dimensionless constant for gain scheduling according to Eq. 32
K_F	—	dimensionless coefficient used in calculating e_F
L	—	dryer length, m
M	—	dry basis moisture content of grain, kg water / kg dry grain
M_{set}	—	set point for the dry basis moisture content of the grain at the outlet of the dryer, kg water / kg dry grain.
$\bar{\bar{M}}_{in}$	—	arithmetic average of the initial moisture contents of all the grain currently in the dryer (dry basis), kg water / kg dry grain
$\bar{M}_{out}(s.s)$	—	average outlet moisture at steady state, kg water / kg dry grain (dry basis)
N_R	—	number of rows into which the dryer is conceptually divided
r	—	dimensionless reset rate in the feedback algorithm
R	—	drying rate, kg water / kg dry grain − s
R_c	—	characteristic drying rate, kg water / kg dry grain − s
S_B	—	dimensionless steady-state gain of the feedback loop defined in Eq. 28a–28b
t	—	time, s
t_j	—	time in the beginning of the j^{th} time interval, s
T	—	air temperature, °C
u	—	dummy variable of integration, dimensionless
W	—	humidity ratio of air, kg water / kg dry air
x	—	coordinate along the latitudinal axis of the dryer, m
y	—	coordinate along the longitudinal axis of the dryer, m
α	—	dimensionless group in the dryer model
γ	—	dimensionless group in the dryer model
δ	—	dimensionless group in the dryer model
ϵ	—	bed porosity, m^3 pores / m^3 total volume
θ	—	grain temperature, °C
ρ	—	density of dry grain, kg/m^3
τ	—	residence time of the grain in the dryer, s
τ_B	—	the feedback correction for the grain residence time determined by the feedforward algorithm, s

τ_F	—	the residence time of the grain according to the feedforward algorithm, s
ϕ	—	dimensionless group in the dryer model
ψ	—	dimensionless group in the dryer model

General Symbols

V_{in}	—	variable V at the inlet of the dryer
$\overline{V}_{in}$	—	spatial average of variable V at the inlet of the dryer
$\overline{V}_{out}$	—	spatial average of variable V at the outlet of the dryer
$\overline{V}_{out}(s.s)$	—	the steady-state spatial average of variable V at the outlet of the dryer
V*	—	dimensionless form of variable V
V(0)	—	variable V at time zero
V(j)	—	variable V in the beginning of the j^{th} time interval
V(i,j)	—	variable V corresponding to the i^{th} row of the dryer in the beginning of the j^{th} time interval
$(V)_i$	—	variable V corresponding to the i^{th} row of the dryer
ΔV	—	a change in variable V

LITERATURE CITED

BAKKER-ARKEMA, F. W., R. G. MOREIRA, and A. Y. ELTIGANI
1989. Automatic control of cross-flow grain dryers. *In* Vincent A. Dodd and Patrick M. Grace (eds.), Proc. of 11th International Congress on Agricultural Engineering/Dublin/4-8 September 1989, 4:2199–2204. A. A. Balkema/Rotterdam/Brookfield.

ELTIGANI, A. Y., and F. W. BAKKER-ARKEMA
1987. Automatic control of commercial cross-flow grain dryers. *Drying Technology* 5(4):561–75.

FORBES, J. F., B. A. JACOBSON, E. RHODES, and G. R. SULLIVAN
1984. Model based control strategies for commercial grain drying systems. *The Canadian J. Chem. Engng.* 62(12):773–79.

GUI, X. Q., J. BENTSMAN, and J. B. LITCHFIELD
1988. Control of grain drying: state of the art and open problems. ASAE Paper No. 88-3502, ASAE, St. Joseph, MI 49085.

HOLTMAN, J. B., and G. L. ZACHARIAH
1969. Computer controls for grain dryers. *Trans. of the ASAE* 12(4):433–37.

MARCHANT, J. A.
1985. Control of high-temperature continuous flow grain dryers. *The Agric. Eng.* 40(4):145–49.

MOREIRA, R. G., and F. W. BAKKER-ARKEMA
1990. A Feedforward/feedback adaptive controller for commercial cross-flow grain dryers. *J. Agric. Engng. Res.* 45:107–16.

NYBRANT, T. G.
1988. Modelling and adaptive control of continuous grain dryers. *J. Agric. Engng Res.* 40:165–73.

NYBRANT, T. G., and P. J. S. REGNÉR
1985. Adaptive control of continuous grain dryers. ASAE Paper No. 85-3011, ASAE, St. Joseph, MI 49085.

PLATT, D., T. R. RUMSEY, and A. PALAZOGLU
1990. Dynamic modelling of a cross-flow rice dryer. *Hilgardia* 58:1–46.

STEPHANOPOULOS, G.
1984. *Chemical Process Control.* Prentice-Hall, Englewood Cliffs, NJ.

WHITFIELD, R. D.
1986. An unsteady-state simulation to study the control of concurrent and counter-flow grain dryers. *J. Agric. Engng. Res.* 33:171–78.

WHITFIELD, R. D.
1988. Control of a mixed-flow drier. *J. Agric. Engng. Res.* 41:275–99.

ZACHARIAH, G. L., and G. W. ISAACS
1966. Simulating a moisture-control system for a continuous-flow drier. *Trans. of the ASAE.* 9(4):297–302.

2m-pr-4/92-LTE/VFG

The Journal HILGARDIA is published irregularly. Number of pages and number of issues vary per annually numbered volume. Address: Agriculture and Natural Resources Publications, University of California, 300 Lakeside Drive, 6th Floor, Oakland, CA 94612-3550.