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A JOURNAL OF AGRICULTURAL SCIENCE PUBLISHED BY
THE CALIFORNIA AGRICULTURAL EXPERIMENT STATION

Volume 62 • Numbers 2,3,4,5 • December 1995

Tree Taper Model Volume Equations

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IV. Tree Volume Equations for Major California Conifers

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UNIVERSITY OF CALIFORNIA DIVISION OF AGRICULTURE AND NATURAL RESOURCES

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ABSTRACT

Upper-stem tree diameters are usually measured outside the bark with the only bark thickness measurements being made at breast height. The current study presents equations and coefficients for estimating the bark thickness in both the upper stem and at the stumps based upon the bark thickness at breast height, the size of the tree (DBH and total height), and the height to the measurement in question.

The upper-stem model presented is an extension of a previously published general hyperbolic ratio model. The model for bark ratio below breast height, a simple power function, is estimated as well. Both equations contain coefficients that account for the more rapid taper on trees with thicker bark.

The data used for fitting and testing the taper models consisted of measurements on over 3,000 conifer trees measured by members of the Northern California Forest Yield Cooperative and the USDA Forest Service. The data were split into two halves, one half for fitting and the other half for testing. The best upper-stem and below-DBH equations obtained from this analysis are discussed here; the remainder are listed in the Appendix.

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Bark Taper Equations for California Conifers[1]

INTRODUCTION

WHILE WE ARE usually interested in the tree diameter inside bark for estimating tree volume, it is usually outside-bark diameters that are observed, often using an optical device to estimate tree diameters at various heights above the ground. This leaves the problem of estimating the bark thickness for each outside-bark diameter measured along the stem. For many of the same conifer species considered here, Khan, Bell, and Berg (1977), Ritchie and Hann (1984), Dolph (1984, 1989), and Larsen and Hann (1985) developed expressions for inside-bark diameter as a function of outside bark diameter. These include linear and nonlinear regressions and ratio models. However, as noted by Assman (1970, p.73), within the same species bark thickness varies "according to site, age, and racial characteristics of the tree." Thus, improvements in the predictions of bark at any height in the tree can be expected with information on the bark thickness at breast height (4.5 feet) of each tree.

We examined alternative models for expressing the relative taper of the bark so that bark thickness observed at breast height can be used to estimate the bark thickness at various points along the stem. The equations considered included hyperbolic ratios by Grosenbaugh (1974) and Brickell (1970), and segmented polynomial models by Maguire and Hann (1990) and Max and Burkhart (1976).[2] This paper presents the best upper-stem and lower-stem (stump) models that resulted from that comparison, while the Appendix lists the models tested.

When measuring standing trees with an optical dendrometer, the usual process is to measure the bark thickness at breast height by either cutting into the stem with an ax or by using a bark gauge at 4.5 feet above the ground. In order to reduce the error in measuring bark thickness, and since bark thickness varies around the stem, typically two such measurements are taken at right angles to each other and added to estimate the double bark thickness at breast height, DBTBH. Bark taper models can then be used to express the reduction in bark thickness as one goes from breast height to the tip or to the stump of the tree. Thus, the measurement of bark thickness at breast height "localizes" the predictions of bark thickness at other points.

The conifer species considered include the following:

Code	Species
PP	ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.)
SP	sugar pine (*P. lambertiana* Dougl.)
DF	Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco)
WF	white fir (*Abies concolor* [Gord. & Glend.] Lindl. ex Hildebr.)
RF	red fir (*A. magnifica* A. Murr.)
IC	incense-cedar (*Calocedrus decurrens* [Torr.] Florin)

[1]Accepted for publication May 30, 1995.

[2]See also earlier work by Fuller (1969), Gallant and Fuller (1973), and Gallant (1974).

EQUATIONS

In this study, available bark thickness models were evaluated for their ability to predict the bark taper from felled tree data collected in five different studies of stem taper in California conifers (see "Data" below). The upper-stem (above breast height) and lower-stem (below breast height) bark taper equations evaluated are listed in Appendix tables A1 and A2. Coefficients and fit statistics for each data set are given in Appendix tables A3 to A6. Only the best model (model 6) is presented here. Each equation is formulated to predict the relative bark thickness and the variables used include the following:

DBH	diameter outside bark at breast height (4.5 feet)
DBT	double bark thickness at height h
DBTBH	double bark thickness at breast height
DIB	diameter inside bark at height h
DOB	diameter outside bark at height h
h	height to measurement
THT	total height of tree

Also, the following ratios are used:

RB	relative bark thickness, $\dfrac{DBT}{DBTBH}$
$\widehat{RB}$	predicted relative bark thickness, RB
RD	relative diameter, $\dfrac{DOB}{DBH}$

We assume that the bark thickness at breast height can be measured.

For the upper stem, the equation

$$\widehat{RB}_u = RD\left(\frac{b_1 - 1}{b_1 - RD^{b_2}}\right) - \left(\frac{RD^{b_3} - 1}{DBTBH}\right) \tag{1}$$

for the variables defined above and the coefficients b_1, b_2, and b_3 were estimated for each species. However, to make equation (1) monotonic, it must be bounded. Thus, we have the conditional function RB'_u as follows

RB'_u is equal to $\widehat{RB}_{min}$ if $RD < \widehat{RD}_{min}$ or equal to $\widehat{RB}_u$ if $RD \geq \widehat{RD}_{min}$

where $\widehat{RD}_{min}$ = diameter ratio at which the minimum bark ratio ($\widehat{RB}_{min}$) occurs. The relationship between these minimum values and DBTBH is estimated by

$$\widehat{RD}_{min} \approx \frac{1}{a * DBTBH} \tag{2}$$

where $\widehat{RD}_{min}$ is the estimated minimum relative diameter for each species as a function of DBTBH and the coefficient a is computed for each species.

For predicting bark ratios below breast height the equation used is

$$RB_s = 1 + c_1\left(\frac{DOB}{DBH} - 1\right)^{(c_2 + c_3\,DBTBH)} \tag{3}$$

where c_1, c_2, and c_3 are coefficients estimated for each species. In other models tested, we observed that the models tended to underestimate the bark ratio in the tips for larger values of DBTBH. Thus, in both equations (1) and (3) the DBTBH component of the equation is designed to eliminate this observed bias.

DATA SOURCES

The bark taper data used here are taken from five separate sources where bark measurements along the tree stem were made as part of separate studies on tree taper or utilization. Each of these data sources is described briefly here and the number of trees in each data set is shown by species in Table 1.

TABLE 1. NUMBER OF TREES BY SPECIES AND DATA SETS

Species & Data Set	Trees
	no.
Ponderosa pine	
COOP	197
DOLPH	141
MILL	177
ALL	515
Sugar pine	
COOP	65
DOLPH	62
MILL	54
ALL	181
Douglas-fir	
COOP	186
DOLPH	44
ALL	230
White fir	
COOP	352
DOLPH	313
MILL	161
FIR STUDY	76
ALL	902
Red fir	
COOP	36
MILL	148
FIR STUDY	562
REGION 6	273
ALL	1019
Incense-cedar	
COOP	81
DOLPH	189
MILL	54
ALL	324

The "Coop" bark data set comes from a stem analysis data set developed by members of the Northern California Forest Yield Cooperative. Formed in 1978, this cooperative research group links the research staff at the University of California with various private forestry companies and public agencies to produce growth-prediction models for California conifers. A previous study (Biging 1984) used the inside-bark tree diameters to develop stem-taper equations for California conifers. Measurements on these same trees were also used for initial growth models for CACTOS, the California Conifer Timber Output Simulator (Wensel, Meerschaert, and Biging, 1987). The data came from industry forest lands in the north and central Sierra Nevada, the southern Cascades, the Shasta-Trinity mountains, and the Mendocino region in California.

The "Dolph" data set comes from a study of conifer trees in California's Sierra Nevada and was collected for the purpose of developing a California PROGNOSIS variant (Dolph 1984 and 1989). The "Fir Study" data set was contributed to this study by Bill Oliver (USFS, Redding) and was the basis for his growth studies on thinned true fir stands. "Region 6" refers to USDA Forest Service data from southern Oregon. These data were collected from sample trees in southern Oregon to adjust the regional gross volume inventory estimates for local utilization.

The "Mill Study" data set comes from a series of utilization studies conducted in California by the USDA Forest Service Pacific Northwest Forest Experiment Station. Sample trees were felled and scaled in the field, scaled again in the mill before being sent through the mill for conversion. Product recovery research results have been published by Ernst and Pong (1985), Pong (1982), and Pong and Cahill (1988) where these data are described in more detail.

The data were received from the various contributors and cooperators and stored in a common data base. The data were then retrieved from this data base using a screening process that eliminated trees and/or observations that were obviously in error or would result from forked, broken, or grossly malformed trees. Since field data sheets for most of the data were not readily available, there were no further changes made to the edited data. The mean, minimum, and maximum tree DBH and total height by species are given in Table 2.

TABLE 2. NUMBER OF TREES, MEAN, MINIMUM, AND MAXIMUM DIAMETER AT BREAST HEIGHT AND TOTAL HEIGHT USED TO FIT BARK TAPER MODELS BY SPECIES

		Diameter			Height		
Species	**Trees**	**Mean**	**Min**	**Max**	**Mean**	**Min**	**Max**
	no.		inches			feet	
Ponderosa pine	515	21.1	5.5	60.2	100.7	18.0	202.0
Sugar pine	181	24.1	3.5	62.5	94.8	9.0	204.0
Douglas-fir	230	15.4	5.5	31.2	86.7	25.5	150.6
White fir	902	17.6	5.5	62.9	80.2	18.6	204.0
Red fir	1019	20.9	3.0	57.4	85.6	10.7	195.0
Incense-cedar	324	13.8	3.4	50.8	49.5	7.4	155.0

RESULTS AND DISCUSSION

The SAS nonlinear fitting procedure, NLIN, was used with the Marquardt option (SAS Institute, 1985). Both the upper-stem and lower-stem (stump) equations were fitted to each of the six major conifer species considered. Table 3 gives the coefficients for the upper stem equations 1 and 2; Table 4 gives the coefficients for the stump, equation (3). Coefficients for other models tested appear in Appendix tables A3 and A4 with fit statistics for all models and data sets in Appendix tables A5 and A6. Equation (3) gave the smallest residual sums of squares for the testing half of all data sets.

The adjusted hyperbola used for the upper stem, using equations (1) and (2), is illustrated for Douglas-fir in Figure 1 for various bark thicknesses at breast height. The power function for the stump bark ratio, using equation (3), is illustrated for Douglas-fir in Figure 2. The addition of an exponent as a function of DBTBH in equation (3) reduced the overall correlation of predicted with residual relative bark ratios.

Our extensive look at conifer bark taper has resulted in the recommendation of the two new models for bark taper presented here. For upper stems the adjusted hyperbolic equation, equtation (1), is recommended; and for stump bark taper, the power equation (3) is recommended. Both represent distinct improvements in the way bark taper estimates have been handled by other models tested (see Appendix). However, these bark taper models only apply to trees with undamaged bark that have bark thickness decreasing from breast height to the tip and bark thickness increasing from breast height down to the stump. Trees that show damage due to fire and/or logging may not follow this pattern.

TABLE 3. COEFFICIENTS FOR UPPER STEM EQUATION (1) AND MINIMUM DIAMETER RATIO EQUATION (2)

	Equation (1)			Equation (2)
Species	b_1	b_2r	b_3	a
Ponderosa pine	2.8532	6.188	.150480	7.787
Sugar pine	4.9396	443.812	.098117	12.503
Douglas-fir	2.8350	13.625	.055268	15.078
White fir	2.6718	5.821	.090351	10.098
Red fir	6.1661	52.944	.095993	13.388
Incense-cedar	3.6087	9.350	.060086	15.883

TABLE 4. COEFFICIENTS CALCULATED FOR STUMP BARK RATION EQUATION (3)

Species	c_1	c_2	c_3
Ponderosa pine	1.73416	0.0*	0.475413
Sugar pine	1.0*	0.0*	0.678624
Douglas-fir	1.0*	0.0*	0.397823
White fir	1.0*	0.0*	0.555289
Red fir	0.880132	0.260076	0.225097
Incense-cedar	1.87591	0.559340	0.171258

*Fixed.

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ACKNOWLEDGMENTS

The authors gratefully acknowledge the assistance of Ralph Warbington, Kevin Casey, Charles Stadelman, and LeRoy Dolph of the U.S. Forest Service in helping to accumulate the data sets used for this study and to Al Stangenberger, Kip Freytag, and Frieder Schurr for help in preparing the data for analysis. The current study is a cooperative effort between the University of California, the USDA Forest Service – Region 5 and Pacific Southwest Research Station, the California Department of Forestry and Fire Protection, Michigan-California Lumber Company, and Crane Mills.

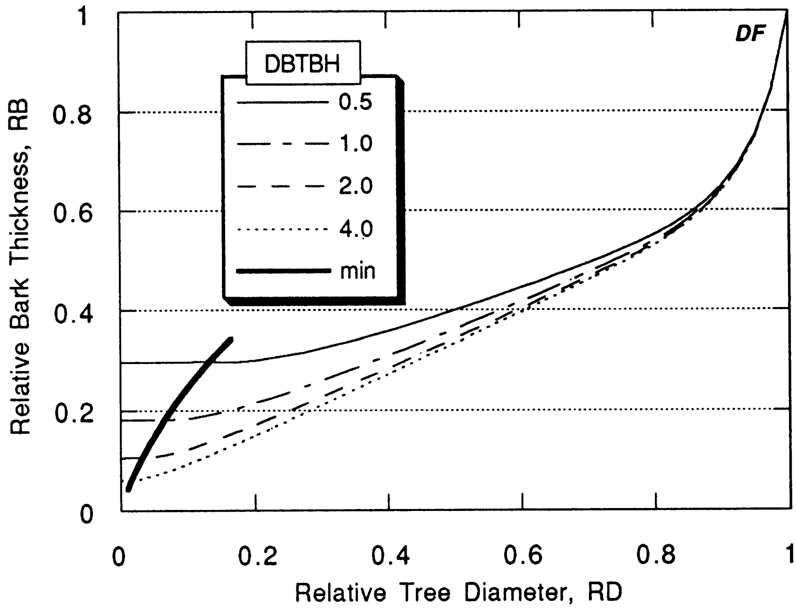

Fig. 1. Relative bark thickness for DBTBH = 0.5, 1.0, 2.0, and 4.0 as a function of relative diameter using equation (1) and minimum of each function from equations (2) and (1)—Douglas-fir.

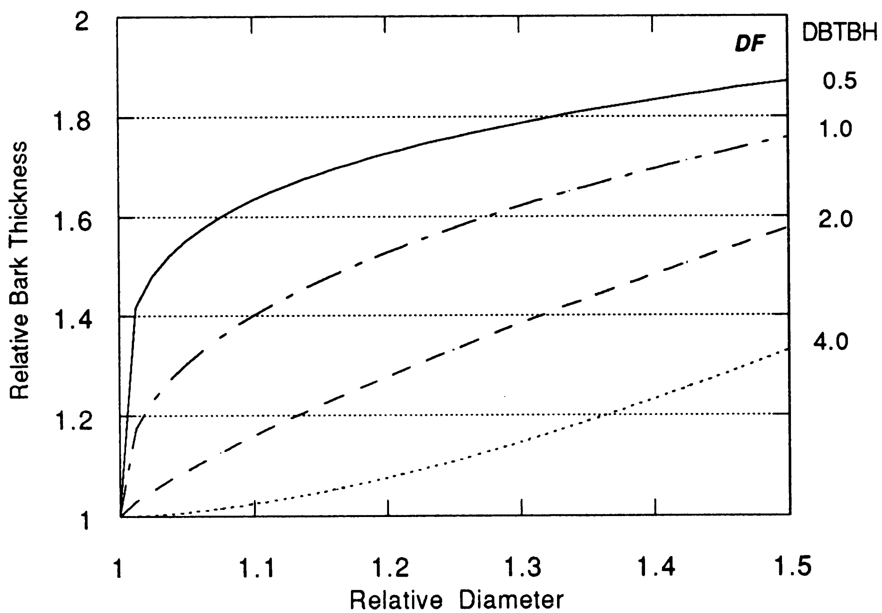

Fig. 2. Relative bark thickness below 4.5 feet for DBTBH = 0.5, 1.0, 2.0, and 4.0 as a function of relative tree diameter—Douglas-fir.

APPENDIX TABLE A1. HYPERBOLIC BARK TAPER EQUATIONS EVALUATED, WHERE Y IS THE PREDICTED BARK RATIO $\frac{DBT}{DBTBH}$.

Double bark thickness is then predicted by multiplying by DBTBH. Variable definitions are given in text.

No. 1 Hyperbolic ratio (Grosenbaugh 1974, option 2)

$$Y_1 = \left(\frac{DOB}{DBH}\right)\left(\frac{1}{2 - \frac{DOB}{DBH}}\right)$$

No. 2 No. 1 with coefficient fitted to data (Bricknell 1970, option 1)

$$Y_2 = \left(\frac{DOB}{DBH}\right)\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)$$

No. 3 Brickell option 3

$$Y_3 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)^{b_3}}\right)^{b_4}$$

No. 4 Brickell option 4

$$Y_4 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)$$

No. 5 Variant between Brickell's options 3 and 4

$$Y_5 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)^{b_4}$$

No. 6 Adjusted hyperbolic ratio (text equation 1)

$$Y_6 = \left(\frac{DOB}{DBH}\right)\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)b_3}\right) - \left(\frac{\left(\frac{DOB}{DBH}\right)^{b_5} - 1}{DBTBH}\right)$$

for DOB > D_{min} and $Y_6 = F_{min}$ otherwise where D_{min} and F_{min} denote the coordinates of the minimum of the above function.

APPENDIX TABLE A2. SEGMENTED POLYNOMIAL BARK TAPER EQUATIONS EVALUATED, WHERE Y IS THE PREDICTED BARK RATIO $\frac{DBT}{DBTBH}$.

Double bark thickness is then predicted by multiplying by DBTBH. Variable definitions are given in text.

No. 7 Localized two-segmented polynomial on relative height (Maguire and Hann 1990). Let $X = \frac{HT}{THT}$ and

$$a_1 = b_1 + b_2\left(\frac{THT}{DBH}\right) + b_3\left(\frac{THT}{DBH}\right)^2$$

then

$$Y_7 = 1 + z_1 + a_1 z_2 + a_2 z_3$$

for

$$z_1 = \left\{\frac{(X-1)}{k-1}\left[1 + \frac{(k-X)}{k-1}\right] - 1\right\}$$

$$z_2 = X + I\left\{\frac{(X-1)}{k-1}\left[X + \frac{k(k-X)}{k-1}\right] - X\right\}$$

$$z_3 = X^2 + I\left\{\frac{k(X-1)}{k-1}\left[2X - k + \frac{k(k-X)}{k-1}\right] - X^2\right\}$$

with k = 0.3 and I = 1 for X^3 k and zero otherwise. The coefficient a_2, b_1, and b_3 to be estimated.

No. 8 Localized two-segmented polynomial on relative diameter model as above but with $X = 1 - \frac{DOB}{DBH}$.

No. 9 Two-segmented polynomial on relative diameter (Max and Burkhart 1976).

where $X = 1 - \frac{DOB}{DBH}$, I_1 for X^2 a_1 (zero otherwise),and a_1, b_1, b_2 and b_3 are coefficients to be estimated.

No. 10 Three-segmented polynomial on relative diameter (Max and Burkhart 1976).

$$Y_{10} = Y_9 + b_4(a_2 - X)^2 I_2$$

where Y_9 is defined above, I_2 = 1 for X^2 a_2 (zero otherwise) and a_2 and b_4 are additional coefficients.

APPENDIX TABLE A3. COEFFICIENTS CALCULATED FROM THE "FIT" HALF OF THE DATA FOR HYPERBOLIC MODELS

Species	Model #	b_1	b_2	b_3	b_4	b_5
Ponderosa pine	2		2.05347*			
	3	.172435*	1.00677*	.029665*	.649123*	
	4	.095474	1.31635			
	5	.349935	1.09045		.388315	
	6		2.85323	6.18840		.150480
Sugar pine	2		3.35449			
	3	−.021007*	1.01705*	.031161*	1.10561*	
	4	.219662	1.46440			
	5	.679999	60.7655*		43.2687*	
	6		4.93961	443.812*		.098117
Douglas fir	2		1.65893			
	3	.721122	8.01472*	11.3072*	3.89232*	
	4	.193696	1.27256			
	5	.614854	1.00847		.150581	
	6		2.83504	13.6250		.055268
White fir	2		1.79431			
	3	.186981*	.989607*	−.062291*	.572898*	
	4	.077587	1.28022			
	5	.250571	1.14080		.575405	
	6		2.67181	5.82073		.090351
Red fir	2		3.46315			
	3	.256430	.944620	−.450786	.313750	
	4	.177066	1.48455			
	5	.641516	42.6754*		31.7995*	
	6		6.16610	52.9442		.095993
Incense-cedar	2		2.43416			
	3	.200062*	.996129*	−.012814*	.715154*	
	4	.381861	1.46319			
	5	.751977	1.00961		.105277	
	6		3.60876	9.34989		.060086

The * above implies that the fitted coefficient is not statistically significant; for b_2, it is not significantly different from 2 and, for the others, they are not different from 1.

APPENDIX TABLE A4. COEFFICIENTS CALCULATED FOR THE "FIT" HALF OF EACH DATA SET FOR THE SEGMENTED POLYNOMIAL MODELS

Species	Model #	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Ponderosa pine	7	-2.83552	-.205678	.017598	5.91209				
	8	-2.68503	-.133710	.011663	4.50627				
	9	.352847				.173569*	-.649957	4.10060	
	10	.174750	.921401			49.8355*	-27.2028*	7.66930	27.5576*
Sugar pine	7	-2.68484	-.185314	.015244	5.56146				
	8	-2.39568	-.080486	.006466	3.75759				
	9	.235572				-.164997	-.501945	5.78076	
	10	.039684	.938852			73.5727*	-39.4965*	115.620*	39.5929*
Douglas-fir	7	-4.34707	.249995	-.012264	5.48281				
	8	-3.94069	.085076	-.002958	5.56035				
	9	.215982				-.155540	-.396021	9.47537	
	10	.185933	.948954			38.3031	-20.5246	10.8073	20.4324
White fir	7	-3.11532	-.077110	.010508	5.30685				
	8	-2.81758	-.143934	.011142	4.85128				
	9	.386240				.387050	-.757016	4.13236	
	10	.226295	.923448			52.2488	-28.4194	5.21484	28.7965
Red fir	7	-2.07456	-.140579	.009701	4.23880				
	8	-1.73861	-.222968	.015331	3.34266				
	9	.206937				-.298995	-.433534	6.05364	
	10	.124518	.941632			55.9460	-30.0884	11.7623	30.0820
Incense-cedar	7	-3.42176	-.163537	.024326	6.03113				
	8	-1.96023	-.293174	.027806	3.47811				
	9	.242376				-.451194	-.235204	5.24302	
	10	.214742	.968996			87.0817	-45.3019	5.56182	45.2831

The * above implies that the fitted coefficient is not statistically different from 1.

APPENDIX TABLE A5. RESIDUAL SUM OF SQUARES (RSS) FOR TEST HALF OF DATA SETS FOR THE HYPERBOLIC BARK MODELS BY SPECIES AND DATA SETS

Hyperbolic Models

Species & Dataset	No. of Obs.	Model # 1	Model # 2	Model # 3	Model # 4	Model # 5	Model # 6
Ponderosa pine							
COOP	1232	42.40	42.10	33.30	33.80	33.30	29.70
DOLPH	836	8.47	8.56	5.54	5.40	5.54	5.05
MILL	673	17.37	17.52	13.91	14.08	13.92	13.07
ALL	2741	68.30	68.22	52.75	53.27	52.73	47.86
Sugar pine							
COOP	424	18.41	14.81	12.53	12.49	13.27	11.02
DOLPH	351	2.53	4.12	2.56	2.73	4.59	3.00
MILL	242	9.89	8.55	7.55	7.65	8.27	6.40
ALL	1017	30.83	27.49	22.64	22.87	26.13	20.95
Douglas-fir							
COOP	1164	27.30	26.50	18.90	20.70	18.90	17.60
DOLPH	237	3.32	2.65	1.47	1.43	1.48	1.41
ALL	1401	30.70	29.14	20.37	22.16	20.41	19.04
White fir							
COOP	2048	64.10	66.20	49.60	50.29	49.60	45.40
DOLPH	1988	24.90	21.90	15.50	15.22	15.50	15.40
MILL	514	11.69	11.34	9.87	9.88	9.87	10.26
FIR STUDY	441	6.70	6.05	4.12	4.12	4.11	3.88
ALL	4991	107.40	105.48	79.09	79.51	79.08	74.91
Red fir							
COOP	178	5.53	4.64	3.34	3.63	4.07	3.23
MILL	452	12.80	9.90	9.10	9.20	9.70	9.15
FIR STUDY	3607	68.80	64.00	44.00	46.80	60.60	42.50
REGION 6	1367	43.80	31.70	29.40	30.40	29.60	30.51
ALL	5604	130.60	110.35	85.84	90.03	104.04	85.38
Incense-cedar							
COOP	355	6.65	6.14	5.66	5.76	5.66	5.26
DOLPH	844	7.89	8.48	6.03	6.16	6.03	4.91
MILL	114	6.78	5.79	5.41	5.69	5.42	5.72
ALL	1313	21.32	20.41	17.10	17.61	17.11	15.89

APPENDIX TABLE A6. RESIDUAL SUM OF SQUARES (RSS) FOR TEST HALF OF DATA SETS FOR THE SEGMENTED POLYNOMIAL BARK MODELS BY SPECIES AND DATA SET

(The hyperbolic model #6 is included for comparison.)

Species & Dataset	Observations (no.)	*Model # 6*	Model # 7	Model # 8	Model # 9	Model # 10
Ponderosa pine						
COOP	1232	*29.70*	38.30	34.70	34.70	33.10
DOLPH	836	*5.05*	11.44	5.61	5.60	5.63
MILL	673	*13.07*	17.78	14.10	14.31	13.88
ALL	2741	*47.86*	67.50	54.40	54.63	52.61
Sugar pine						
COOP	424	*11.02*	14.46	12.90	12.77	11.67
DOLPH	351	*3.00*	6.92	2.88	2.93	3.73
MILL	242	*6.40*	7.48	7.11	7.14	7.07
ALL	1017	*20.95*	28.87	22.88	22.84	22.46
Douglas-fir						
COOP	1164	*17.60*	19.40	19.40	19.30	19.10
DOLPH	237	*1.41*	3.36	1.39	1.43	1.38
ALL	1401	*19.04*	22.75	20.75	20.76	20.48
White fir						
COOP	2048	*45.40*	59.20	52.40	52.00	49.80
DOLPH	1988	*15.40*	27.30	16.20	15.90	15.60
MILL	514	*10.26*	11.55	9.37	9.89	9.83
FIR STUDY	441	*3.88*	4.66	4.19	4.20	4.12
ALL	4991	*74.91*	102.70	82.07	81.99	79.29
Red fir						
COOP	178	*3.23*	3.96	3.57	3.49	3.30
MILL	452	*9.15*	9.72	9.10	9.20	9.10
FIR STUDY	3607	*42.50*	48.99	44.60	45.60	44.50
REGION 6	1367	*30.51*	37.07	28.28	29.30	29.20
ALL	5604	*85.38*	99.76	85.56	87.59	86.07
Incense-cedar						
COOP	355	*5.26*	6.99	5.83	5.70	5.64
DOLPH	844	*4.91*	12.93	5.75	6.10	6.06
MILL	114	*5.72*	6.17	4.94	5.33	5.35
ALL	1313	*15.89*	26.10	16.51	17.13	17.05